Day 11 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Lift and cable tension (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A lift of mass 300 kg carries a person of mass 80 kg. The cable tension is 4028 N and the lift is accelerating. Find (a) the magnitude and direction of the acceleration, (b) the force of the floor on the person. Take g = 9.8.

Whole system (mass 380 kg), taking upwards as positive: $T - 380\,g = 380\,a$

$4028 - 3724 = 380\,a$, so $a = 0.8$

(a) a is positive, so the acceleration is 0.8 m s^{-2} upwards

(b) Person: $R - 80\,g = 80\,a$, so $R = 80(9.8 + 0.8)$

Answer: (a) 0.8 m s^{-2} upwards (b) 848 N

2 Indices: solving equations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $27^{x+1} = 3^{2x-1}$.

Write both sides as powers of 3.

$(3^3)^{x+1} = (3^1)^{2x-1}$

$3(x + 1) = 2x - 1$

$3x + 3 = 2x - 1$

$x = -4$

Answer: $x = -4$

3 Continuous random variable: finding k and E(X) (CIE)

The random variable X has probability density function $f(x) = k(5 - x)$ for $0 \leq x \leq 5$, and 0 otherwise.

(a) Find k. (b) Find E(X). (c) Find $P(X > \frac{10}{3})$.

(a) $\int_0^5 f(x)\,dx = 1$ gives $\frac{25}{2}k = 1$, so $k = \frac{2}{25}$

(b) $E(X) = \int x\,f(x)\,dx = \frac{5}{3}$

(c) $P(X > \frac{10}{3}) = \int_{\frac{10}{3}}^5 f(x)\,dx = \frac{1}{9}$

Answer: (a) $k = \frac{2}{25}$ (b) $\frac{5}{3}$ (c) $\frac{1}{9}$

4 Product rule: stationary points (All boards)

$y = x^3\,e^{-3x}$ for $x > 0$.

Find the exact coordinates of the stationary point and determine its nature.

$\frac{dy}{dx} = 3x^2 \times e^{-3x} + x^3 \times (-3)e^{-3x}$

$= x^2(3 - 3x)e^{-3x}$

$e^{-3x} > 0$ and $x > 0$, so $\frac{dy}{dx} = 0$ gives $3 - 3x = 0$ and $x = 1$

$y = e^{-3}$; $\frac{d^2y}{dx^2}$ is negative there, so it is a maximum

Answer: $(1, e^{-3})$, maximum

5 Polynomial division (All boards)

Find the quotient and the remainder when $3x^3 + 6x^2 - 4x + 20$ is divided by $(x + 3)$.

Quotient (matching leading terms in turn): $3x^2 - 3x + 5$

$(x + 3)(3x^2 - 3x + 5) = 3x^3 + 6x^2 - 4x + 15$

Remainder $= 3x^3 + 6x^2 - 4x + 20 - (3x^3 + 6x^2 - 4x + 15) = 5$

Answer: quotient $3x^2 - 3x + 5$, remainder 5

6 Regression with coded variables (AQA, Edexcel, OCR, MEI, Eduqas)

A regression line of v on w is $v = -2965 + -15w$, where $w = \frac{t - 1990}{10}$.

(a) Find the regression line of v on t in the form $v = p + qt$.
(b) Estimate v when $t = 2020$.

$v = -2965 + -15 \times \frac{t - 1990}{10}$

$= -2965 + -1.5t - -2985 = 20 + -1.5t$

(b) $v = 20 + -1.5 \times 2020 = -3010$

Answer: (a) $v = 20 - 1.5t$ (b) -3010

7 Recurrence relations: limits (All boards)

$u_1 = 1$ and $u_{n+1} = \frac{1}{3} u_n + 6$.

(a) Find the limit L of the sequence as $n \to \infty$.
(b) Find the least n for which $|u_n - L| < 0.1$.

(a) $L = \frac{1}{3} L + 6$ gives $L = \frac{6}{1 - \frac{1}{3}} = 9$

(b) $u_n - L = (u_1 - L)\, r^{n-1} = (-8) \times \frac{1}{3}^{n-1}$

$|-8| \times \frac{1}{3}^{n-1} < \frac{1}{10}$

Check: n = 4 gives 0.29630, n = 5 gives 0.09877

Answer: (a) $L = 9$ (b) $n = 5$

8 Separable differential equations (All boards)

Solve the differential equation

$y\frac{dy}{dx} = x$,

given that $y = 1$ when $x = 0$ ($y > 0$). Give y in terms of x.

Separate: $\int y\, dy = \int x\, dx$, so $\frac{y^2}{2} = \frac{1}{2}x^2 + C$

$x = 0$, $y = 1$ gives $C = \frac{1}{2}$

$y^2 = x^2 + 1$

Answer: $y = \sqrt{x^2 + 1}$

9 Smooth incline held by a string (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A 4 kg particle rests on a smooth plane inclined at α, $\tan \alpha = \frac{4}{3}$, held by a string in the vertical plane through a line of greatest slope, at angle θ above the slope, $\tan \theta = \frac{5}{12}$. Find the tension and the normal reaction. Take $g = 9.8$. Give answers to 3 s.f. if not exact.

$\sin \alpha = 0.8$, $\cos \alpha = 0.6$, $\sin \theta = \frac{5}{13}$, $\cos \theta = \frac{12}{13}$

Along the plane: $T \cos \theta = 4\, g \sin \alpha$, so $T = 34.0$

Perpendicular: $R + T \sin \theta = 4\, g \cos \alpha$

$R = 4\, g \cos \alpha - T \sin \theta = 23.5 - 34.0 \times 0.385$

Answer: $T = 34.0$ N, $R = 10.5$ N

10 Differentiating sec, cosec, tan and cot (All boards)

y = tan(2x).

Find $\frac{dy}{dx}$ and its exact value at x = $\frac{\pi}{6}$.

$\frac{dy}{dx} = 2\sec^2(2x)$

At x = $\frac{\pi}{6}$ the angle 2x = $\frac{\pi}{3}$, so the gradient is 8

Answer: $\frac{dy}{dx} = 2\sec^2(2x)$; 8