Day 109 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Connected rates of change: spheres, circles and cubes (AQA, Edexcel, OCR, MEI, Eduqas)

The radius of a sphere increases at 1 cm s^{-1}.
Find the rate of increase of its volume when r = 4 cm, in terms of π.

$V = \frac{4}{3}\pi r^3$, so $\frac{dV}{dr} = 4\pi r^2$

$\frac{dV}{dt} = \frac{dV}{dr} \times \frac{dr}{dt} = 4\pi \times 4^2 \times 1 = 64\pi$

Answer: 64π cm^3/s

2 Systematic sampling (AQA, Edexcel, OCR, MEI, Eduqas)

A systematic sample is taken from a numbered list of 100 items.
The 3th item chosen is number 11 and the 6th item chosen is number 26.
Find the sampling interval, the number of the first item chosen and the size of the sample.

Interval = (26 − 11) ÷ (6 − 3) = 5
First item = 11 − 2 × 5 = 1
Sample size = 100 ÷ 5 = 20
Answer: Interval 5, first 1, size 20

3 Sigma notation (All boards)

Find the value of $\sum_{r=1}^{5} 2 \times \frac{2}{3}^r$.

Geometric series: first term $\frac{4}{3}$, ratio $\frac{2}{3}$, 5 terms

$S = \frac{\frac{4}{3}(\frac{2}{3}^5 - 1)}{(\frac{2}{3} - 1)}$

$= \frac{\frac{4}{3} \times -\frac{211}{243}}{-\frac{1}{3}}$

Answer: $\frac{844}{243}$

4 Connected particles on a rough incline (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The system is released from rest and A moves up the rough plane. Find (a) the acceleration, (b) the tension in the light string. Take g = 9.8. Give answers to 3 s.f. if not exact.

sin α = 0.8, cos α = 0.6; friction F = μR = 0.25×3 g cos α
B: 11 g − T = 11 a
A: T − 3 g sin α − F = 3 a
Adding: g(11 − 2.4 − 0.45) = 14 a
(a) a = 5.705; (b) T = 11(g − a)
Answer: (a) 5.705 m s^{-2} (b) 45.0 N

5 Laws of logarithms in terms of p and q (All boards)

Given that $\log_a 2 = p$ and $\log_a 3 = q$,
express $\log_a(\sqrt{54})$ in terms of p and q.

$\sqrt{54} = 2^{\frac{1}{2}} \times 3^{\frac{3}{2}}$

$\log_a(2^{\frac{1}{2}} \times 3^{\frac{3}{2}}) = \frac{1}{2}\log_a 2 + \frac{3}{2}\log_a 3$

$= \frac{1}{2}p + \frac{3}{2}q$

Answer: $\frac{1}{2}p + \frac{3}{2}q$

Day 109 **A Level Maths: A Tier 10-a-day: Model Solutions**

6 Power, driving force and resistance (CIE)

A 2000 kg car has engine power 30 kW and constant resistance 600 N. Find (a) its greatest steady speed on a horizontal road, (b) its acceleration at 12 m s^{-1} up a hill where $\sin\alpha = \frac{1}{20}$. Take g = 10.

(a) Steady speed: driving force = resistance. P = Fv: 30000 = 600 v
(b) Driving force F = P ÷ v = 30000 ÷ 12 = 2500 N
Up the hill: F − 600 − 2000 g sin α = 2000 a
2500 − 600 − 1000 = 2000 a
a = 0.45
Answer: (a) 50 m s^{-1} (b) 0.45 m s^{-2}

7 Circle: tangent at a point (AQA, Edexcel, OCR, MEI, Eduqas)

The circle $x^2 + y^2 - 4x + 2y - 95 = 0$ passes through P(−6, −7).
(a) Find the equation of the tangent at P, in the form ax + by = c.
(b) Find the coordinates of the point where this tangent crosses the x-axis.

Centre (2, −1). Gradient of radius CP = $\frac{3}{4}$

Gradient of tangent = $-\frac{4}{3}$

$y + 7 = -\frac{4}{3}(x + 6)$: $4x + 3y = -45$

y = 0: $x = -\frac{45}{4}$

Answer: (a) 4x + 3y = −45 (b) $(-\frac{45}{4}, 0)$

8 Dividing a line in a ratio (2D position vectors) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A and B have position vectors $\begin{pmatrix}6\\5\end{pmatrix}$ and $\begin{pmatrix}3\\5\end{pmatrix}$. The point P lies on AB with AP : PB = 1 : 2.

Find the position vector of P and the exact distance OP.

$\overrightarrow{OP} = \overrightarrow{OA} + \frac{1}{3}\overrightarrow{AB} = \begin{pmatrix}6\\5\end{pmatrix} + \frac{1}{3}\begin{pmatrix}-3\\0\end{pmatrix}$

$= \begin{pmatrix}5\\5\end{pmatrix}$

$OP = \sqrt{5^2 + 5^2} = \sqrt{50} = 5\sqrt{2}$

Answer: $\begin{pmatrix}5\\5\end{pmatrix}$, OP = $5\sqrt{2}$

9 Conditional binomial probability (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

X ~ B(10, 0.2). Find P(X = 7 | X ≥ 4) to 3 significant figures.

$P(X = 7 \mid X \geq 4) = \frac{P(X = 7)}{P(X \geq 4)}$ (since X = 7 implies X ≥ 4)
P(X = 7) = 0.000786
P(X ≥ 4) = 1 − P(X ≤ 3) = 0.121
Ratio = 0.00651
Answer: 0.00651

10 Partial fractions: repeated factor (All boards)

Express $\frac{3x^2 + 9}{(x + 3)(x - 3)^2}$ in the form $\frac{A}{(x + 3)} + \frac{B}{(x - 3)} + \frac{C}{(x - 3)^2}$, and find A, B and C.

$3x^2 + 9 \equiv A(x - 3)^2 + B(x + 3)(x - 3) + C(x + 3)$
x = −3: $36 = A(-6)^2$ so A = 1
x = 3: 36 = C(6) so C = 6
Coefficient of x^2: 3 = A + B so B = 2
Answer: A = 1, B = 2, C = 6