Day 108 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Completing the square (All boards)

$f(x) = 3x^2 - 30x + 66$.

(a) Write f(x) in the form $a(x + p)^2 + q$.

(b) State the minimum value of f(x) and the value of x at which it occurs.

(c) Solve f(x) = 0, giving surds in simplest form.

$f(x) = 3(x - 5)^2 - 9$

(b) minimum value -9 when $x = 5$

(c) $3(x - 5)^2 = 9$, so $(x - 5)^2 = 3$

$x - 5 = \pm\sqrt{3}$

Answer: (a) $3(x - 5)^2 - 9$ (b) minimum -9 at $x = 5$ (c) $x = 5 \pm \sqrt{3}$

2 Differentiating sec, cosec, tan and cot (All boards)

$y = \text{cosec}(3x)$.

Find $\frac{dy}{dx}$ and its exact value at $x = \frac{\pi}{9}$.

$\frac{dy}{dx} = -3\text{cosec}(3x)\cot(3x)$

At $x = \frac{\pi}{9}$ the angle $3x = \frac{\pi}{3}$, so the gradient is -2

Answer: $\frac{dy}{dx} = -3\text{cosec}(3x)\cot(3x)$; -2

3 Lift and cable tension (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A lift of mass 500 kg carries a person of mass 75 kg. The cable tension is 5347.5 N and the lift is accelerating. Find (a) the magnitude and direction of the acceleration, (b) the force of the floor on the person. Take g = 9.8.

Whole system (mass 575 kg), taking upwards as positive: $T - 575g = 575a$

$5347.5 - 5635 = 575a$, so $a = -0.5$

(a) a is negative, so the acceleration is 0.5 m s^{-2} downwards

(b) Person: $R - 75g = 75a$, so $R = 75(9.8 - 0.5)$

Answer: (a) 0.5 m s^{-2} downwards (b) 697.5 N

4 Systematic sampling (AQA, Edexcel, OCR, MEI, Eduqas)

The 540 items in a list are numbered 1 to 540.

A systematic sample of 36 items is taken, starting with a random item among the first few.

The first item selected is number 6.

Find the sampling interval and the number of the last item selected.

Interval = $540 \div 36 = 15$

Items selected: 6, 21, 36, ...

Last = $6 + (36 - 1) \times 15 = 531$

Answer: Interval 15, last item 531

5 Perpendicular bisector and area (All boards)

A is $(-5, 3)$ and B is $(5, -3)$.

(a) Find the equation of the perpendicular bisector of AB, in the form $ax + by = c$.

(b) The perpendicular bisector meets the x-axis at P. Find the area of triangle ABP.

Midpoint of AB = (0, 0); gradient AB = $-\frac{3}{5}$

Perpendicular gradient = $\frac{5}{3}$

Equation: $5x - 3y = 0$

$y = 0$: P = (0, 0)

Area = $\frac{1}{2}|(5 - (-5))(0 - 3) - (0 - (-5))(-3 - 3)|$

Answer: (a) $5x - 3y = 0$ (b) 0

6 Normal approximation to the binomial (All boards)

X ~ B(120, 0.3). Use a normal approximation to estimate P(X ≤ 42) to 3 significant figures.

np = 36, npq = 25.2, so Y ~ N(36, 25.2)
With continuity correction: P(Y < 42.5)
$z = \frac{42.5 - 36}{5.02} = 1.29$, P(Z < 1.29) = 0.902

Answer: 0.902

7 Greatest and least values using R-form (All boards)

$g(x) = \frac{1}{11 + 6\sin x + 8\cos x}$

(a) Write 6 sin x + 8 cos x in the form R sin(x + α), with α acute.
(b) Find the greatest and least values of g(x) and the smallest positive x at which the greatest value occurs (1 d.p.).

$R = \sqrt{6^2 + 8^2} = 10$, $\alpha = \tan^{-1}\frac{8}{6} = 53.1°$

Denominator = 11 + 10 sin(x + 53.1°), ranging from 1 to 21

Greatest g = $\frac{1}{1}$ (denominator least), least g = $\frac{1}{21}$

Least denominator when sin(x + 53.1°) = −1: x + 53.1° = 270°
x = 216.9°

Answer: R = 10, α = 53.1°; greatest $\frac{1}{1}$, least $\frac{1}{21}$; x = 216.9°

8 Connected particles on a rough incline (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The system is released from rest and A moves up the rough plane. Find (a) the acceleration, (b) the tension in the light string. Take g = 9.8. Give answers to 3 s.f. if not exact.

sin α = 0.6, cos α = 0.8; friction F = μR = 0.2×4 g cos α
B: 4 g − T = 4 a
A: T − 4 g sin α − F = 4 a
Adding: g(4 − 2.4 − 0.64) = 8 a
(a) a = 1.176; (b) T = 4(g − a)

Answer: (a) 1.176 m s^{-2} (b) 34.5 N

9 Range of a composite function (All boards)

$f(x) = x^2 - 6x + 11$, $x \in \mathbb{R}$, and $g(x) = \frac{4}{x - 1}$, x > 1.

(a) Find the range of f.
(b) Show that gf is defined and find its range.

$f(x) = (x - 3)^2 + 2 \geq 2$
So f(x) ≥ 2 > 1, which lies in the domain of g, so gf is defined.

$gf(x) = \frac{4}{f(x) - 1}$, with f(x) ≥ 2, so f(x) − 1 ≥ 1

Largest value is $\frac{4}{1} = 4$ at the minimum of f; gf(x) > 0.

Answer: (a) f(x) ≥ 2 (b) 0 < gf(x) ≤ 4

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10 Optimisation problems (All boards)

A closed cylinder has radius r cm and volume 250π cm^3.

Show that its surface area is $A = 2\pi r^2 + \frac{500\pi}{r}$, and find the minimum value of A in terms of π.

$h = \frac{250}{r^2}$, so $A = 2\pi r^2 + 2\pi rh = 2\pi r^2 + \frac{500\pi}{r}$

$\frac{dA}{dr} = 4\pi r - \frac{500\pi}{r^2} = 0$, so $r^3 = 125$ and $r = 5$

$\frac{d^2A}{dr^2} = 4\pi + \frac{1000\pi}{r^3} > 0$, so a minimum

$A = 2\pi \times 25 + \frac{500\pi}{5} = 150\pi$

Answer: A_min = 150π cm^2 (r = 5)