**Day 107** **A Level Maths: A Tier 10-a-day: Model Solutions**

## 1 Circle: centre, radius and position of a point (AQA, Edexcel, OCR, MEI, Eduqas)

A circle has equation $x^2 + y^2 + 12x + 10y + 36 = 0$.
(a) Find the centre and radius of the circle.
(b) Determine whether P(−4, −3) lies inside or outside the circle.

Complete the square: $(x + 6)^2 + (y + 5)^2 = 25$
Centre (−6, −5), radius 5
Distance$^2$ from centre to P = $2^2 + 2^2 = 8$
Compare with $r^2 = 25$
**Answer: (a) centre (−6, −5), radius 5 (b) P is inside the circle**

## 2 Projectile from and to the ground (AQA, Edexcel, OCR, MEI, Eduqas)

A ball is projected from a point O on horizontal ground at 24.5 m s$^{-1}$ at angle α above the horizontal, $\tan\alpha = \frac{4}{3}$, and lands at A. Find (a) the time of flight, (b) the greatest height, (c) OA. Take g = 9.8.

Horizontal component = 24.5×0.6 = 14.7; vertical component = 24.5×0.8 = 19.6
(a) Vertical: $0 = 19.6\,t - 4.9t^2$, so t = 4
(b) At the top: $0 = 19.6^2 - 2\times9.8\times H$, so H = 19.6
(c) OA = 14.7 × 4
**Answer: (a) 4 s (b) 19.6 m (c) 58.8 m**

## 3 Algebraic fractions (All boards)

Simplify fully $\frac{4x^2 + 6x - 10}{2x^2 + 2x - 4}$.

Factorise the numerator: (2x − 2)(2x + 5)
Factorise the denominator: (2x − 2)(x + 2)
Cancel the common factor (2x − 2).
**Answer:** $\frac{2x + 5}{x + 2}$

## 4 Partial fractions and integration (All boards)

(a) Express $\frac{-x - 1}{(x + 2)^2}$ in the form $\frac{A}{x + 2} + \frac{B}{(x + 2)^2}$.
(b) Hence find the exact value of $\int_0^3 \frac{-x - 1}{(x + 2)^2}\,dx$.

$-x - 1 \equiv A(x + 2) + B$, so A = −1 and B = 1
$I = [-1\ln(x + 2) - \frac{1}{x + 2}]_0^3$
$= -1\ln\frac{5}{2} + 1(\frac{1}{2} - \frac{1}{5})$
$= \frac{10\ln 2 + 3 - 10\ln 5}{10}$
**Answer: (a) A = −1, B = 1; (b)** $\frac{10\ln 2 + 3 - 10\ln 5}{10}$

## 5 Stopping distance (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A driver at constant speed sees a hazard 70.4 m ahead. After a reaction time of 0.4 s she brakes with deceleration 2 m s$^{-2}$ and stops exactly at the hazard. Find (a) her initial speed, (b) the total time from seeing the hazard to stopping.

Let the initial speed be v. Reaction distance = 0.4 v, braking distance = $v^2 \div (2\times2)$
$0.4\,v + v^2 \div 4 = 70.4$, so $v^2 + 1.6\,v - 281.6 = 0$
(v − 16)(v + 17.6) = 0, so v = 16
(b) Braking time = 16 ÷ 2 = 8 s; total = 0.4 + 8
**Answer: (a) 16 m s$^{-1}$ (b) 8.4 s**

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## 6 Hypothesis test for correlation (AQA, Edexcel, OCR, MEI, Eduqas)

For a sample of 8 pairs of values of hours of sunshine and plant height, $r = -0.839$.
The critical value for a two-tailed test at the 5% level is 0.7067.
Test, at this level, whether there is any correlation between the two variables.

$H_0: \rho = 0$, $H_1: \rho \neq 0$
Critical value 0.7067
$|r| = 0.839 > 0.7067$
This is in the critical region, so reject $H_0$.
There is evidence of negative correlation.
**Answer: Reject $H_0$: evidence of correlation**

## 7 Tangent conditions (All boards)

Find the set of values of k for which the line $y = x + k$ does not meet the curve $y = x^2 + 4x - 4$.

$x^2 + 4x - 4 = x + k$ gives $x^2 + 3x + (-4 - k) = 0$
Discriminant: $3^2 - 4 \times 1(-4 - k) = 25 + 4k$
No intersection: discriminant $< 0$
**Answer: $k < -\frac{25}{4}$**

## 8 Connected rates of change: cones and bowls (AQA, Edexcel, OCR, MEI, Eduqas)

Water pours into a cone, vertex down, at 50 $cm^3 s^{-1}$. The surface radius is always $\frac{h}{3}$.

Find the rate at which the depth is rising when h = 2 cm.

$V = \frac{1}{3}\pi(h/3)^2 h = \frac{\pi h^3}{27}$, so $\frac{dV}{dh} = \frac{\pi h^2}{9}$
$\frac{dh}{dt} = \frac{dV}{dt \div dV/dh} = \frac{50 \times 9}{\pi h^2}$
$h = 2$: $\frac{dh}{dt} = \frac{225}{2\pi}$ cm $s^{-1}$
**Answer: $\frac{225}{2\pi}$ cm/s**

## 9 Binomial in two stages (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Eggs are defective with probability 0.2, independently. A tray of 5 eggs is rejected if it has at least 2 defective.
From 11 such trays, find the probability that exactly 4 are rejected (3 s.f.).

$Y \sim B(5, 0.2)$. $P(\text{rejected}) = P(Y \geq 2) = 1 - 0.737 = 0.263$
$W \sim B(11, 0.263)$ is the number rejected
$P(W = 4) = C(11, 4)(0.263)^4(1 - 0.263)^7 = 0.186$
**Answer: 0.186**

## 10 Parametric differentiation with trigonometry (All boards)

A curve has parametric equations $x = 2\sec t$, $y = \tan t$.

Find $\frac{dy}{dx}$ in terms of t and the exact equation of the tangent at the point where $t = \frac{\pi}{4}$, in the form $y = mx + c$.

$\frac{dy}{dx} = \frac{dy}{dt \div dx/dt} = \frac{1}{2\sin t}$
$t = \frac{\pi}{4}$: gradient $m = \frac{\sqrt{2}}{2}$, point $(2\sqrt{2}, 1)$
$c = y - mx = -1$
**Answer: $\frac{dy}{dx} = \frac{1}{2\sin t}$; $y = \frac{\sqrt{2}}{2}x - 1$**