Day 106 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Arithmetic sequences: finding a and d (All boards)

The 3rd term of an arithmetic sequence is -14 and the 7th term is -22.
Find the first term and common difference, and the sum of the first 30 terms.

$u_7 - u_3 = 4\,d = -22 - (-14)$ so $d = -2$

$a + 2\,d = -14$ so $a = -10$

$S_{30} = \frac{30}{2}(2 \times (-10) + 29 \times (-2))$

Answer: $a = -10$, $d = -2$, $S_{30} = -1170$

2 Conditional probability and independence (All boards)

P(B) = 0.15, P(A) = 0.65 and P(A | B) = 0.5.
(a) Find $P(A \cap B)$ and $P(A \cup B)$.
(b) Find P(B | A).
(c) Determine whether A and B are independent.

(a) $P(A \cap B) = P(A \mid B)P(B) = 0.5 \times 0.15 = 0.075$
$P(A \cup B) = 0.65 + 0.15 - 0.075 = 0.725$

(b) $P(B \mid A) = \frac{P(A \cap B)}{P(A)} = \frac{0.075}{0.65} = \frac{3}{26}$

(c) $P(A)P(B) = 0.65 \times 0.15 = \frac{39}{400}$, which does not equal $P(A \cap B) = 0.075$

So A and B are not independent

Answer: (a) 0.075, 0.725 (b) $\frac{3}{26}$ (c) not independent

3 Arc length and sector area (All boards)

A sector of a circle has area 11.25 cm^2 and angle 0.9 radians.
Find the radius, the arc length and the perimeter of the sector.

Area $= \frac{1}{2}r^2\theta$: $11.25 = \frac{1}{2}r^2 \times 0.9$, so $r^2 = 25 = 5^2$

$r = 5$ cm; arc length $s = r\theta = 5 \times 0.9 = 4.5$ cm
Perimeter $= 2r + s = 10 + 4.5 = 14.5$ cm

Answer: $r = 5$ cm, arc 4.5 cm, perimeter 14.5 cm

4 Circle: centre, radius and position of a point (AQA, Edexcel, OCR, MEI, Eduqas)

A circle has equation $x^2 + y^2 - 6x + 4y = 0$.
(a) Find the centre and radius of the circle.
(b) Determine whether P(2, 2) lies inside or outside the circle.

Complete the square: $(x - 3)^2 + (y + 2)^2 = 13$

Centre $(3, -2)$, radius $\sqrt{13}$

Distance^2 from centre to P $= (-1)^2 + 4^2 = 17$

Compare with $r^2 = 13$

Answer: (a) centre $(3, -2)$, radius $\sqrt{13}$ (b) P is outside the circle

5 Connected particles (rough table, string slack) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Particle A (5 kg) lies on a rough table, $\mu = 0.2$, joined by a light string over a smooth pulley to particle B (3 kg), 1 m above the floor. Released from rest; B lands without rebounding. Find (a) the acceleration of A before B lands, (b) the further distance A moves. Take $g = 9.8$.

B: $3g - T = 3a$; A: $T - 0.2{\times}5g = 5a$

Adding: $(3 - 0.2{\times}5)g = 8a$, so $a = 2.45$

Speed of B and A as B lands: $v^2 = 2{\times}2.45{\times}1 = 4.9$

After B lands, A decelerates at $\mu g = 0.2{\times}9.8 = 1.96$

$0 = v^2 - 2(\mu g)s$, $s = 4.9 \div 3.92$

Answer: (a) 2.45 m s^{-2} (b) 1.25 m

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6 Area and perimeter of a segment (exact) (All boards)

The circle has centre O and radius 4 cm. Angle AOB = $\frac{\pi}{2}$.

Find the exact area and the exact perimeter of the shaded segment.

Sector area = $\frac{1}{2}r^2\theta = 4\pi$; triangle OAB = $\frac{1}{2}r^2\sin\theta = 8$

Segment area = sector − triangle = $4\pi - 8$

Chord AB = $2r\sin\frac{\theta}{2} = 4\sqrt{2}$; arc = $r\theta = 2\pi$

Perimeter = arc + chord = $4\sqrt{2} + 2\pi$

Answer: area = $4\pi - 8$ cm^2, perimeter = $4\sqrt{2} + 2\pi$ cm

7 Two balls meeting (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Ball A is dropped from rest 73.5 m above the ground. At the same instant ball B is projected upwards at 24.5 m s^{-1} from the ground directly below A. Find (a) the time when they collide, (b) the height of the collision. Take g = 9.8.

Let the time be t. Distance fallen by A: $\frac{1}{2}gt^2 = 4.9t^2$
Height of B above the ground: $24.5\,t - 4.9t^2$
They meet when the distances add up to 73.5: $4.9t^2 + (24.5\,t - 4.9t^2) = 73.5$
so $24.5\,t = 73.5$, $t = 3$
Height of B = $24.5\times3 - 4.9\times3^2 = 29.4$
Answer: (a) 3 s (b) 29.4 m

8 Inverse trigonometric functions (AQA, Edexcel, OCR, MEI, Eduqas)

Find the exact value of $\sin(\arcsin\frac{8}{17} + \arcsin\frac{7}{25})$.

Let A = $\arcsin\frac{8}{17}$ and B = $\arcsin\frac{7}{25}$, both acute

$\cos A = \frac{15}{17}$, $\cos B = \frac{24}{25}$

$\sin(A + B) = \sin A\cos B + \cos A\sin B = \frac{8}{17}\times\frac{24}{25} + \frac{15}{17}\times\frac{7}{25} = \frac{297}{425}$

Answer: $\frac{297}{425}$

9 Binomial expansion: finding n and k (All boards)

In the expansion of $(1 + kx)^n$, where n is a positive integer, the coefficient of x is 9 and the coefficient of x^2 is 36. Find n and k.

$nk = 9$ and $\frac{n(n-1)}{2}k^2 = 36$

$(n-1)k = \frac{2\times36}{9} = 8$ (dividing the second equation by the first)

$n\,k - (n-1)\,k = k = 9 - 8 = 1$

$n = \frac{9}{1} = 9$

Answer: n = 9, k = 1

10 Normal then binomial (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The mass of an egg is N(77, 8^2). An egg is classed as 'special' if its mass is more than 86. 8 eggs are chosen at random. Find the probability that exactly 4 are special (3 s.f.).

P(special) = P(X > 86) = P(Z > 1.12) = 0.130
Y ~ B(8, 0.130)
P(Y = 4) = C(8, 4)$(0.130)^4(0.870)^4$ = 0.0115
Answer: 0.0115