Day 105 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Constant acceleration (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A cyclist passes P with speed 7 m s^{-1} and accelerates uniformly. She travels 300 m in the next 20 s, reaching Q. Find (a) her acceleration, (b) her speed at Q, (c) her speed at the midpoint of PQ.

(a) $s = ut + \frac{1}{2}at^2$: $300 = 7\times20 + \frac{1}{2}\times a\times20^2$, so $a = 0.8$
(b) $v = u + at = 7 + 0.8\times20 = 23$
(c) For the midpoint, $s = 150$: $v^2 = u^2 + 2as = 7^2 + 2\times0.8\times150$
$v^2 = 289$
Answer: (a) 0.8 m s^{-2} (b) 23 m s^{-1} (c) 17 m s^{-1}

2 Indices: solving equations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $5^{x+2} = 25^{2x-3}$.

Write both sides as powers of 5.
$(5^1)^{x+2} = (5^2)^{2x-3}$
$x + 2 = 2(2x - 3)$
$x + 2 = 4x - 6$
$-3x = -8$
Answer: $x = \frac{8}{3}$

3 Venn diagrams and conditional probability (All boards)

P(A) = 0.6, P(B) = 0.2 and $P(A \cap B) = 0.05$.
Find
(a) $P(A \cup B)$
(b) $P(B' \mid A')$
(c) $P(A \mid B)$

(a) $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.6 + 0.2 - 0.05 = 0.75$
(b) $P(B' \mid A') = \frac{P(A' \cap B')}{P(A')} = \frac{1 - 0.75}{1 - 0.6} = 0.625$
(c) $P(A \mid B) = \frac{P(A \cap B)}{P(B)} = \frac{0.05}{0.2} = 0.25$
Answer: (a) 0.75 (b) 0.625 (c) 0.25

4 Line meets curve: length of chord (All boards)

The line $y = -2x + 1$ meets the curve $y = x^2 - 4x - 14$ at A and B.
(a) Find the coordinates of A and B.
(b) Find the exact length AB.

$x^2 - 4x - 14 = -2x + 1$ gives $x^2 - 2x - 15 = 0$
$(x + 3)(x - 5) = 0$: $x = -3$ or 5
A(−3, 7), B(5, −9)
$AB^2 = 8^2 + (-16)^2 = 320$
Answer: A(−3, 7), B(5, −9); AB = $8\sqrt{5}$

5 Logarithmic equations (All boards)

Solve $\log_2(x + 7) + \log_2(x - 8) = 4$.

Combine: $\log_2[(x + 7)(x - 8)] = 4$, so $(x + 7)(x - 8) = 2^4 = 16$
$x^2 - x - 72 = 0$: $(x - 9)(x + 8) = 0$
$x = 9$ or $x = -8$; $x = -8$ makes $x - 8$ negative, so it is rejected
Answer: x = 9

6 Two balls meeting (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Ball A is dropped from rest 49 m above the ground. At the same instant ball B is projected upwards at 24.5 m s^{-1} from the ground directly below A. Find (a) the time when they collide, (b) the height of the collision. Take g = 9.8.

Let the time be t. Distance fallen by A: $\frac{1}{2}gt^2 = 4.9t^2$
Height of B above the ground: $24.5\,t - 4.9t^2$
They meet when the distances add up to 49: $4.9t^2 + (24.5\,t - 4.9t^2) = 49$
so $24.5\,t = 49$, $t = 2$
Height of B = $24.5 \times 2 - 4.9 \times 2^2 = 29.4$
Answer: (a) 2 s (b) 29.4 m

7 Exponential growth and decay modelling (All boards)

A car costs £18000 new. Its value falls by 10% each year, so after t years its value is £V.
(a) Write down V in terms of t. (b) Find, to 3 s.f., the time when the value is £6000.

$V = 18000 \times 0.9^t$

$18000 \times 0.9^t = 6000$ gives $0.9^t = \frac{1}{3}$

$t = \frac{\ln(1/3)}{\ln 0.9} = 10.4$ years

Answer: (a) $V = 18000 \times 0.9^t$ (b) t = 10.4 years

8 Binomial in two stages (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Eggs are defective with probability 0.1, independently. A tray of 9 eggs is rejected if it has at least 3 defective. From 12 such trays, find the probability that exactly 3 are rejected (3 s.f.).

$Y \sim B(9, 0.1)$. $P(\text{rejected}) = P(Y \geq 3) = 1 - 0.947 = 0.0530$
$W \sim B(12, 0.0530)$ is the number rejected
$P(W = 3) = C(12, 3)(0.0530)^3(1 - 0.0530)^9 = 0.0200$
Answer: 0.0200

9 Implicit differentiation: horizontal tangents (All boards)

A curve has equation $x^2 - xy + 3y^2 = 11$.
Find the coordinates of the points where the tangent is parallel to the x-axis.

$\frac{dy}{dx} = -\frac{2x - y}{-x + 6y}$
Horizontal tangent: $2x - y = 0$, so $y = 2x$
Substitute: $11x^2 = 11$, so $x^2 = 1$ and $x = \pm 1$
Points: (1, 2) and (−1, −2)
Answer: (1, 2) and (−1, −2)

10 Binomial expansion: approximating a square root (All boards)

(a) Expand $(1 - 2x)^{\frac{1}{2}}$ in ascending powers of x up to x^2.
(b) Use x = 0.01 to estimate $\sqrt{98}$ to 4 decimal places.

$(1 + y)^{\frac{1}{2}} = 1 + \frac{1}{2}y - \frac{1}{8}y^2$, $y = -2x$

(a) $1 - x - \frac{1}{2}x^2$

$x = \frac{1}{100}$: $98 = 10^2(1 - \frac{1}{50})$ so $\sqrt{98} = 10 \times (1 - \frac{1}{50})^{\frac{1}{2}}$

$\approx 10 \times (1 - \frac{1}{100} - \frac{1}{20000}) = 9.899500$

Answer: (a) $1 - x - \frac{1}{2}x^2$ (b) 9.8995