Day 104 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Laws of logarithms in terms of p and q (All boards)

Given that $\log_a 2 = p$ and $\log_a 3 = q$,
express $\log_a(36)$ in terms of p and q.

$36 = 2^2 \times 3^2$
$\log_a(2^2 \times 3^2) = 2\log_a 2 + 2\log_a 3$
$= 2p + 2q$
Answer: $2p + 2q$

2 Connected particles (smooth table) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Particle A (7 kg) rests on a smooth table, joined by a light string over a smooth pulley at the edge to particle B (1 kg), hanging 0.5 m above the floor. Released from rest. Find (a) the tension, (b) the speed of B as it hits the floor. Take g = 9.8. Give answers to 3 s.f. if not exact.

B: $1g - T = 1a$
A: $T = 7a$
Adding: $1g = 8a$, so $a = 1.225$
(a) $T = 7 \times 1.225 = 8.575$
(b) $v^2 = 0 + 2 \times 1.225 \times 0.5 = 1.225$
Answer: (a) 8.575 N (b) 1.11 m s^{-1}

3 Algebraic fractions (All boards)

Express $\frac{5}{(x+1)} - \frac{1}{(x-1)}$ as a single fraction in its simplest form.

Common denominator $(x+1)(x-1)$
Numerator: $5(x-1) - 1(x+1) = 4x - 6$
Answer: $\frac{4x-6}{(x+1)(x-1)}$

4 Speed-time graphs (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the speed-time graph for a train journey of 1890 m. Find (a) V, (b) the deceleration in the final stage.

(a) Distance = area of trapezium = ½(120 + 90) × V
1890 = 105 V, so V = 18
(b) Deceleration = V ÷ 15 = 18 ÷ 15
Answer: (a) V = 18 (b) 1.2 m s^{-2}

5 Parametric differentiation with trigonometry (All boards)

A curve has parametric equations $x = 3\cos t$, $y = 3\sin t$.

Find $\frac{dy}{dx}$ in terms of t and the exact equation of the tangent at the point where $t = \frac{\pi}{6}$, in the form $y = mx + c$.

$\frac{dy}{dx} = \frac{dy}{dt \div dx/dt} = -\cot t$

$t = \frac{\pi}{6}$: gradient $m = -\sqrt{3}$, point $(\frac{3\sqrt{3}}{2}, \frac{3}{2})$

$c = y - mx = 6$

Answer: $\frac{dy}{dx} = -\cot t$; $y = -\sqrt{3}x + 6$

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6 Rationalising the denominator (AQA, Edexcel, OCR, MEI, Eduqas)

Express $\frac{4 - 3\sqrt{3}}{5 + 3\sqrt{3}}$ in the form $p + q\sqrt{3}$, where p and q are rational numbers.

Multiply top and bottom by $5 - 3\sqrt{3}$.
Denominator: $(5 + 3\sqrt{3})(5 - 3\sqrt{3}) = -2$
Numerator: $(4 - 3\sqrt{3})(5 - 3\sqrt{3}) = 47 - 27\sqrt{3}$
So the expression is $\frac{47 - 27\sqrt{3}}{-2}$.

Answer: $\mathbf{-\frac{47}{2} + \frac{27}{2}\sqrt{3}}$

7 Normal approximation to the Poisson (CIE)

X ~ Po(40). Use a suitable approximation to find P(X ≥ 48), to 3 significant figures.

λ > 15, so Y ~ N(40, 40) approximately
Continuity correction: P(Y > 47.5)
z = 1.19, so probability = 0.118
Answer: 0.118

8 Exponential growth and decay modelling (All boards)

A radioactive sample decays so that its mass is $M = M_0e^{-kt}$ grams, with t in days.

Its half-life is 5 days. Show that $k = \frac{\ln 2}{5}$ and find how long it takes for 90% of the sample to decay.

Half-life: $e^{-5k} = \frac{1}{2}$, so $5k = \ln 2$ and $k = \frac{\ln 2}{5}$
90% decayed means 10% remains: $e^{-kt} = 0.1$
$kt = -\ln 0.1 = \ln 10$, so $t = \frac{5 \ln 10}{\ln 2} = 16.6$ days
Answer: t = 16.6 days

9 Binomial in two stages (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Parts are defective with probability 0.05, independently. A box of 8 parts is rejected if it has at least 2 defective. From 12 such boxes, find the probability that exactly 2 are rejected (3 s.f.).

Y ~ B(8, 0.05). P(rejected) = P(Y ≥ 2) = 1 − 0.943 = 0.0572
W ~ B(12, 0.0572) is the number rejected
$P(W = 2) = C(12, 2)(0.0572)^2(1 - 0.0572)^{10} = 0.120$
Answer: 0.120

10 Partial fractions and binomial expansion (All boards)

$f(x) = \frac{-12x - 6}{(1 + x)(1 + 3x)}$.

(a) Express f(x) in partial fractions.
(b) Hence expand f(x) up to x^2.
(c) State the range of x for which the expansion is valid.

(a) $f(x) = \frac{A}{(1 + x)} + \frac{B}{(1 + 3x)}$: A = −3, B = −3
(b) $-3(1 + x)^{-1} = -3(1 - x + 1x^2 ...)$
Add the series for both fractions.
(c) Need |x| < 1 and |3x| < 1
Answer: (a) $\mathbf{\frac{-3}{(1 + x)} + \frac{-3}{(1 + 3x)}}$ **(b)** $\mathbf{-6 + 12x - 30x^2}$ **(c)** $\mathbf{|x| < \frac{1}{3}}$