Day 102 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Index equations solved with logarithms (All boards)

Solve $5^{x-2} = 3^{x+3}$.
Give your answer to 3 significant figures.

Take logs: $(x - 2)\ln 5 = (x + 3)\ln 3$
Collect x terms: $x(\ln 5 - \ln 3) = 3\ln 3 + 2\ln 5$
$x = \frac{3\ln 3 + 2\ln 5}{\ln 5 - \ln 3}$
$x = 12.8$
Answer: x = 12.8

2 Binomial probabilities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The probability that a ticket wins a prize is 0.3. In 18 independent trials, find the probability that this happens
(a) exactly 4 times (b) fewer than 4 times (c) at least 4 times.

Let X ~ B(18, 0.3)
(a) P(X = 4) = 0.168
(b) P(X ≤ 3) = 0.165
(c) 1 − P(X ≤ 3) = 0.835
Answer: (a) 0.168 (b) 0.165 (c) 0.835

3 Circle with a given diameter (AQA, Edexcel, OCR, MEI, Eduqas)

A(1, 11) and B(1, −15) are the ends of a diameter of a circle.
(a) Find the equation of the circle, in the form $x^2 + y^2 + ax + by + c = 0$.
(b) Show that C(13, 3) lies on the circle.

Centre = midpoint of AB = (1, −2)
$r^2 = (0)^2 + (13)^2 = 169$
$(x - 1)^2 + (y + 2)^2 = 169$: $x^2 + y^2 - 2x + 4y - 164 = 0$
(b) $CM^2 = (12)^2 + (5)^2 = 169 = r^2$
Answer: (a) $x^2 + y^2 - 2x + 4y - 164 = 0$

4 Projectile from and to the ground (AQA, Edexcel, OCR, MEI, Eduqas)

A ball is projected from a point O on horizontal ground at 27 m s^{-1} at angle α above the horizontal, $\tan\alpha = \frac{5}{12}$, and lands at A. Find (a) the time of flight, (b) the greatest height, (c) OA. Take g = 9.8. Give answers to 3 s.f. if not exact.

Horizontal component = $27\times\frac{12}{13} = \frac{324}{13}$; vertical component = $27\times\frac{5}{13} = \frac{135}{13}$
(a) Vertical: $0 = \frac{135}{13}t - 4.9t^2$, so $t = \frac{1350}{637}$
(b) At the top: $0 = \left(\frac{135}{13}\right)^2 - 2\times9.8\times H$, so $H = \frac{91125}{16562}$
(c) $OA = \frac{324}{13}\times\frac{1350}{637}$
Answer: (a) 2.12 s (b) 5.50 m (c) 52.8 m

5 Small angle approximations: solving an equation (AQA, Edexcel, OCR, MEI, Eduqas)

θ is small and measured in radians. Use small angle approximations to show that
$2\sin 3\theta + 2\cos\theta = 5.5$
can be written as $a\theta^2 + b\theta + c = 0$ with integers a, b, c. Hence find the approximate solution with $0 < \theta < 0.8$, to 3 s.f.

$\sin 3\theta \approx 3\theta$, $\cos\theta \approx 1 - \frac{1}{2}\theta^2$
$2\times 3\theta + 2(1 - \frac{1}{2}\theta^2) = 5.5$
Multiply by 2 and rearrange: $2\theta^2 - 12\theta + 7 = 0$
$\theta = \frac{12 \pm \sqrt{88}}{4}$, so $\theta = 0.655$
Answer: a = 2, b = −12, c = 7; θ ≈ 0.655

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6 Projected up a rough incline (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle is projected at 9.8 m s^{-1} up a line of greatest slope of a rough plane inclined at α, where $\tan\alpha = \frac{4}{3}$. The coefficient of friction is 0.3. Find (a) the greatest distance it moves up the plane, (b) its speed on returning to the start. Take g = 9.8. Give answers to 3 s.f. if not exact.

Up: deceleration = $g(\sin\alpha + \mu\cos\alpha) = 9.8(0.8 + 0.3\times0.6) = 9.604$

(a) $0 = 9.8^2 - 2\times9.604\times d$, so $d = 5$

Down: $\tan\alpha = \frac{4}{3} > \mu$, so it slides back; acceleration = $g(\sin\alpha - \mu\cos\alpha) = 6.076$

(b) $v^2 = 2\times6.076\times5 = 60.76$

$v = \sqrt{60.76}$

Answer: (a) 5 m (b) 7.79 m s^{-1}

7 Venn diagram with algebra (All boards)

The Venn diagram shows the numbers of 32 students in sets A and B.

(a) Find the value of x.

(b) A student is chosen at random. Find P(A | B′).

(a) Total: $(x + 5) + (x + 1) + (x + 4) + 4 = 32$

$3x + 14 = 32$, so $x = 6$

(b) Regions: 11, 7, 10, 4

Not in B: 11 + 4 = 15, of which in A: 11

Answer: (a) x = 6 (b) $\frac{11}{15}$

8 Parametric equations: Cartesian form (AQA, Edexcel, OCR, MEI, Eduqas)

A curve has parametric equations $x = 4 + 2\cos\theta$, $y = -2 + 2\sin\theta$.

Find its Cartesian equation and describe the curve.

$\cos\theta = \frac{x-4}{2}$, $\sin\theta = \frac{y+2}{2}$

Use $\cos^2\theta + \sin^2\theta = 1$: $(x-4)^2 + (y+2)^2 = 4$

Answer: $(x-4)^2 + (y+2)^2 = 4$, a circle with centre $(4, -2)$, radius 2

9 Double and triple angle values (All boards)

θ is obtuse and $\sin\theta = \frac{7}{25}$.

Find the exact values of $\sin2\theta$, $\cos2\theta$ and $\tan2\theta$.

$\cos\theta = -\sqrt{1 - \left(\frac{7}{25}\right)^2} = -\frac{24}{25}$ (obtuse)

$\sin2\theta = 2\sin\theta\cos\theta = 2\times\frac{7}{25}\times\left(-\frac{24}{25}\right) = -\frac{336}{625}$

$\cos2\theta = 1 - 2\sin^2\theta = 1 - 2\times\left(\frac{7}{25}\right)^2 = \frac{527}{625}$

$\tan2\theta = \frac{\sin2\theta}{\cos2\theta} = -\frac{336}{527}$

Answer: $\sin2\theta = -\frac{336}{625}$, $\cos2\theta = \frac{527}{625}$, $\tan2\theta = -\frac{336}{527}$

10 Circle and line: tangent condition (AQA, Edexcel, OCR, MEI, Eduqas)

The line $12x + 5y = k$ is a tangent to the circle $(x-3)^2 + (y+4)^2 = 9$.

Find the two possible values of k.

Centre $(3, -4)$, radius 3

Distance from centre to the line = radius:

$\frac{|16 - k|}{13} = 3$

$|16 - k| = 39$

Answer: $k = -23$ or $k = 55$