Day 101 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Venn diagrams and conditional probability (All boards)

P(A) = 0.65, P(B) = 0.1 and P(A ∩ B) = 0.05.
Find
(a) P(A ∪ B)
(b) P(B | A)
(c) P(A′ ∩ B′)

(a) $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.65 + 0.1 - 0.05 = 0.7$

(b) $P(B \mid A) = \frac{P(A \cap B)}{P(A)} = \frac{0.05}{0.65} = \frac{1}{13}$

(c) $P(A' \cap B') = 1 - P(A \cup B) = 1 - 0.7 = 0.3$

Answer: (a) 0.7 (b) $\frac{1}{13}$ (c) 0.3

2 Friction: find μ then stopping distance (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A 15 kg box is pulled from rest along rough horizontal ground by a horizontal force of 85.5 N and moves 40 m in 10 s. Find (a) the coefficient of friction. The force is then removed; (b) find the further distance moved. Take g = 9.8. Give answers to 3 s.f. if not exact.

$s = \frac{1}{2}at^2$: $40 = \frac{1}{2}a \times 10^2$, so a = 0.8
(a) P − μ15 g = 15 a: 85.5 − 147 μ = 12, so μ = 0.5
(b) v = at = 0.8×10 = 8; after removal deceleration = μg = 4.9
$0 = 8^2 - 2 \times 4.9 \times s$, s = 6.53

Answer: (a) μ = 0.5 (b) 6.53 m

3 Arithmetic sequences: finding a and d (All boards)

The 6th term of an arithmetic sequence is 28 and the 14th term is 76.
Find the first term and common difference, and the sum of the first 20 terms.

$u_{14} - u_6 = 8d = 76 - 28$ so d = 6
a + 5 d = 28 so a = −2

$S_{20} = \frac{20}{2}(2 \times (-2) + 19 \times 6)$

Answer: a = −2, d = 6, S_{20} = 1100

4 Forces in equilibrium with vectors (AQA, Edexcel, OCR, MEI, Eduqas)

Forces $F_1 = (6\mathbf{i} + 2\mathbf{j})$ N, $F_2 = (12\mathbf{i} - 9\mathbf{j})$ N and $F_3 = (a\mathbf{i} + b\mathbf{j})$ N keep a 3 kg particle in equilibrium. Find (a) a and b. F_2 is then removed; (b) find the magnitude of the acceleration.

(a) Equilibrium: $F_1 + F_2 + F_3 = 0$
i: 6 + 12 + a = 0, so a = −18
j: 2 + (−9) + b = 0, so b = 7
(b) The resultant is now $F_1 + F_3 = -F_2$, of magnitude $\sqrt{(12^2 + -9^2)} = 15$ N
a = 15 ÷ 3

Answer: (a) a = −18, b = 7 (b) 5 m s^{-2}

5 Inverse of a quadratic function (All boards)

$f(x) = x^2 + 8x + 18$, $x \geq -4$.
Find $f^{-1}(x)$ and state its domain.

$f(x) = (x + 4)^2 + 2$
Let $y = f(x)$: $(x + 4)^2 = y - 2$
$x \geq -4$, so take the positive root: $x = -4 + \sqrt{y - 2}$

Answer: $f^{-1}(x) = -4 + \sqrt{x - 2}$, $x \geq 2$

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6 Equations with sec x and tan x (All boards)

Solve $2\sec^2\theta + 8\tan\theta + 4 = 0$
for $0° \le \theta \le 360°$. Give answers to 1 d.p.

Use $\sec^2\theta \equiv 1 + \tan^2\theta$: $2\tan^2\theta + 8\tan\theta + 6 = 0$
$\tan\theta = -3$ or $\tan\theta = -1$
$\tan\theta = -3$: $\theta = 108.4°, 288.4°$
$\tan\theta = -1$: $\theta = 135.0°, 315.0°$
Answer: θ = 108.4°, 135.0°, 288.4°, 315.0°

7 Equation of a reciprocal curve from its graph (All boards)

The diagram shows the curve $y = \frac{k}{x - a} + b$ with asymptotes as shown. Find the values of k, a and b, and the coordinates where the curve crosses the axes.

Asymptotes $x = 2$, $y = 2$ so $a = 2$, $b = 2$

Through $(1, -1)$: $-1 = \frac{k}{-1} + 2$

$k = (-1 - 2)(-1) = 3$

$x = 0$: $y = \frac{1}{2}$; $y = 0$: $x = \frac{1}{2}$

Answer: k = 3, a = 2, b = 2; $(0, \frac{1}{2})$ and $(\frac{1}{2}, 0)$

8 Integration by parts (All boards)

Use integration by parts to find the exact value of

$\int_0^{\frac{\pi}{4}} x\cos 2x\,dx$.

$u = x$, $\frac{dv}{dx} = \cos 2x$: $\int x\cos 2x\,dx = [\frac{x\sin 2x}{2}] - \int\frac{\sin 2x}{2}\,dx$

$= [\frac{x\sin 2x}{2} + \frac{\cos 2x}{4}]$

$= \frac{\pi - 2}{8}$

Answer: $\frac{\pi - 2}{8}$

9 Combining two data sets (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The masses in grams of two groups: factory A has n = 28, mean 57, sd 4;
factory B has n = 20, mean 33, sd 2.
Find the mean and standard deviation of all 48 values (3 s.f. for the sd).

$\Sigma x = 28 \times 57 + 20 \times 33 = 2256$, mean = 47

Σx^2 (A) $= 28(4^2 + 57^2) = 91420$

Σx^2 (B) $= 20(2^2 + 33^2) = 21860$

Variance $= \frac{113280}{48} - 47^2 = 151$, so sd = 12.3

Answer: mean 47, sd 12.3

10 Partial fractions and integration (All boards)

(a) Express $\frac{2x+5}{(x+4)^2}$ in the form $\frac{A}{x+4}+\frac{B}{(x+4)^2}$.

(b) Hence find the exact value of $\int_0^4 \frac{2x+5}{(x+4)^2}\,dx$.

$2x + 5 \equiv A(x + 4) + B$, so $A = 2$ and $B = -3$

$I = [2\ln(x+4) - \frac{-3}{x+4}]_0^4$

$= 2\ln\frac{8}{4} + -3(\frac{1}{4} - \frac{1}{8})$

$= \frac{16\ln 2 - 3}{8}$

Answer: (a) $A = 2$, $B = -3$; (b) $\frac{16\ln 2 - 3}{8}$