Day 100 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Projected up a rough incline (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle is projected at 7 m s^{-1} up a line of greatest slope of a rough plane inclined at α, where $\tan\alpha = \frac{12}{5}$. The coefficient of friction is 0.1. Find (a) the greatest distance it moves up the plane, (b) its speed on returning to the start. Take g = 9.8. Give answers to 3 s.f. if not exact.

Up: deceleration = $g(\sin\alpha + \mu\cos\alpha) = 9.8(\frac{12}{13} + 0.1\times\frac{5}{13}) = \frac{245}{26}$

(a) $0 = 7^2 - 2\times\frac{245}{26}\times d$, so d = 2.6

Down: $\tan\alpha = \frac{12}{5} > \mu$, so it slides back; acceleration = $g(\sin\alpha - \mu\cos\alpha) = \frac{1127}{130}$

(b) $v^2 = 2\times\frac{1127}{130}\times 2.6 = 45.08$

$v = \sqrt{45.08}$

Answer: (a) 2.6 m (b) 6.71 m s^{-1}

2 Parametric equations: points on axes (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A curve has parametric equations $x = 3t^2 + 5$, $y = -2t(t^2 - 1)$.
Find the coordinates of the points where the curve meets the x-axis.

y = 0: t = 0 or $t^2 = 1$, so t = 0, 1, −1
t = 0: x = 5
t = ±1: x = 8
Answer: (5, 0) and (8, 0)

3 Differentiating sec, cosec, tan and cot (All boards)

y = x tan(x).

Find the exact gradient at $x = \frac{\pi}{4}$.

Product rule: $\frac{dy}{dx} = \tan(x) + x(\sec^2(x))$

At $x = \frac{\pi}{4}$: tan(x) = 1 and the derivative term = 2

Gradient = $1 + \frac{\pi}{4} \times 2 = \frac{\pi + 2}{2}$

Answer: gradient $\frac{\pi + 2}{2}$

4 Line meets curve: length of chord (All boards)

The line $y = -x - 4$ meets the curve $y = x^2 + 2x - 8$ at A and B.
(a) Find the coordinates of A and B.
(b) Find the exact length AB.

$x^2 + 2x - 8 = -x - 4$ gives $x^2 + 3x - 4 = 0$
$(x + 4)(x - 1) = 0$: x = −4 or 1
A(−4, 0), B(1, −5)
$AB^2 = 5^2 + (-5)^2 = 50$
Answer: A(−4, 0), B(1, −5); AB = $5\sqrt{2}$

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5 Angle between vectors (3D) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The points A, B and C have coordinates (2, −3, −1), (3, 1, 1) and (−3, −1, −1).
Find angle ABC, in degrees to 1 d.p.

BA = $\begin{pmatrix}-1\\-4\\-2\end{pmatrix}$, BC = $\begin{pmatrix}-6\\-2\\-2\end{pmatrix}$

BA·BC = 18

|BA| = $\sqrt{21}$, |BC| = $\sqrt{44}$

cos ABC = $\frac{18}{\sqrt{21}\times\sqrt{44}}$ = 0.5922, so angle ABC = 53.7°

Answer: 53.7°

6 Proof by exhaustion (AQA, Edexcel, OCR, MEI, Eduqas)

Prove that no square number ends in the digit 2, 3, 7 or 8.

The last digit of n^2 depends only on the last digit of n.
Last digit of n: 0 1 2 3 4 5 6 7 8 9
Last digit of n^2: 0 1 4 9 6 5 6 9 4 1
The possible last digits are 0, 1, 4, 5, 6, 9 only.
Answer: last digit of a square is 0, 1, 4, 5, 6 or 9

7 Integration by parts (All boards)

Use integration by parts to find the exact value of

$\int_0^1 xe^{3x}\,dx$.

u = x, $\frac{dv}{dx} = e^{3x}$: $\int xe^{3x}\,dx = [\frac{xe^{3x}}{3}] - \int\frac{e^{3x}}{3}\,dx$

$= [\frac{xe^{3x}}{3} - \frac{e^{3x}}{9}]$

$= \frac{2e^3+1}{9}$

Answer: $\mathbf{\frac{2e^3+1}{9}}$

8 Three-set Venn diagram counts (All boards)

Of 40 students, 14 study Maths (M), 23 Physics (P) and 20 Chemistry (C).
9 study M and P, 7 study M and C, 11 study P and C, and 5 study all three.
One student is chosen at random. Given that a student studies Chemistry, find the probability that they study all three.

Venn regions: M only 3, P only 8, C only 7,
MP only 4, MC only 2, PC only 6, all three 5, outside all 5
Studies Chemistry: 20
Studies all three: 5

Probability = $\frac{5}{20}$

Answer: $\mathbf{\frac{1}{4}}$

9 Maximum speed from velocity function (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle moves in a straight line with velocity v = $(-3t^2 + 18t)$ m s^{-1} at time t s, t ≥ 0, starting at O. Find (a) its greatest speed, (b) its distance from O when t = 6, (c) the time at which it returns to O.

(a) dv/dt = 18 − 6t = 0, so t = 3 (a maximum since the t^2 coefficient is negative)
v = 18×3 − 3×3^2 = 27
(b) x = ∫v dt = 9 t^2 − 1 t^3
x(6) = 108
(c) x = 0 when t^2(9 − 1 t) = 0, so t = 9
Answer: (a) 27 m s^{-1} (b) 108 m (c) 9 s

10 Type I and Type II errors (AQA, Edexcel, OCR, MEI, Eduqas)

$X \sim B(35, p)$. H_0: $p = 0.4$ is tested against H_1: $p < 0.4$ using a 5% significance level.

(a) Find the critical region. (b) State the probability of a Type I error.

(c) Find the probability of a Type II error when $p = 0.25$ (3 s.f.).

(a) Largest/smallest region with probability ≤ 0.05 under H_0: $X \leq 8$

(b) Type I error = P(reject H_0 | $p = 0.4$) = 0.0260

(c) Type II error = P(not in critical region | $p = 0.25$)

$P(X \geq 9)$ when $p = 0.25$: 0.526

Answer: (a) $X \leq 8$ (b) 0.0260 (c) 0.526