Day 1 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Normal approximation to the binomial (All boards)

X ~ B(60, 0.25). Use a normal approximation to estimate P(19 ≤ X ≤ 23) to 3 significant figures.

np = 15, npq = 11.25, so Y ~ N(15, 11.25)
With continuity correction: P(18.5 < Y < 23.5)
z-values 1.04 and 2.53, so probability = 0.143
Answer: 0.143

2 Modulus graph and a line (All boards)

The diagram shows the graphs of y = |3x + 2| and y = −x + 4.
The graphs meet at A and B.
Find the coordinates of A and B.

|3x + 2| = −x + 4
Solve 3x + 2 = ±(−x + 4).
x = −3 and x = $\frac{1}{2}$ (both checked in the original equation)
y = 7 and y = $\frac{7}{2}$
Answer: A(−3, 7), B($\frac{1}{2}$, $\frac{7}{2}$)

3 Discrete random variable with unknown constant (Edexcel, OCR, MEI, Eduqas, CIE)

The random variable X has P(X = x) = kx^2 for x = 2, 3, 4, 5.

(a) Show that k = $\frac{1}{54}$. (b) Find E(X) and Var(X).

(c) Find P(X ≥ 3).

(a) Σ P(X = x) = 1: 4 + 9 + 16 + 25 = 54, so 54k = 1
(b) E(X) = $2 \times \frac{2}{27} + 3 \times \frac{1}{6} + 4 \times \frac{8}{27} + 5 \times \frac{25}{54} = \frac{112}{27}$
$E(X^2) = \frac{163}{9}$, so $Var(X) = \frac{163}{9} - (\frac{112}{27})^2 = \frac{659}{729}$
(c) $P(X \geq 3) = \frac{1}{6} + \frac{8}{27} + \frac{25}{54} = \frac{25}{27}$
Answer: (b) E(X) = $\frac{112}{27}$, Var(X) = $\frac{659}{729}$ (c) $\frac{25}{27}$

4 Fixed point iteration (All boards)

The equation $x^3 = 3x + 4$ has a root α. Use the iteration $x_{n+1} = \sqrt[3]{3x + 4}$ with $x_0 = 1$
to find x_1, x_2 and x_3 to 4 d.p., and hence the value of α to 2 d.p. (continue until the 2 d.p. value is settled).

$x_1 = \sqrt[3]{3 \times 1 + 4} = 1.9129$
$x_2 = 2.1355$, $x_3 = 2.1832$
Continuing, the values settle at 2.1958, so α = 2.20
Answer: x_1 = 1.9129, x_2 = 2.1355, x_3 = 2.1832; α = 2.20

5 Pulley, string goes slack (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Particles A (3 kg) and B (2 kg) hang on a light string over a smooth pulley, A being 2.5 m above the ground. Released from rest; A lands without rebounding. Find (a) the tension while both move, (b) the greatest height of B above its start. Take g = 9.8.

A: 3 g − T = 3 a; B: T − 2 g = 2 a
a = 1 g ÷ 5 = 1.96, so T = 2(g + a) = 23.52
When A lands, B has risen 2.5 m with $v^2 = 2 \times 1.96 \times 2.5 = 9.8$
B then moves freely under gravity: extra height = $v^2 \div (2 \times 9.8) = 0.5$
Total = 2.5 + 0.5
Answer: (a) 23.52 N (b) 3 m

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6 Binomial expansion: product with a bracket (All boards)

Find the coefficient of x^3 in the expansion of
$(2 - 2x)(1 + 2x)^8$.

In $(1 + 2x)^8$: coefficient of x^2 is 112, of x^3 is 448
Coefficient = $2 \times 448 + (-2) \times 112$
Answer: 672

7 Volumes of revolution (CIE (9709); Further Maths on other boards)

The region bounded by the curve $y = \frac{1}{x}$, the x-axis, $x = 1$ and $x = 5$ is rotated through 360° about the x-axis.
Find the exact volume of the solid formed.

$y^2 = \frac{1}{x^2}$, so $V = \pi\int_1^5 y^2\,dx$

$= \pi \times \frac{4}{5}$

$V = \frac{4\pi}{5}$

Answer: $\frac{4\pi}{5}$

8 Binomial expansion: (a + bx)^n for rational n (All boards)

Expand $(16 + x)^{\frac{1}{2}}$ in ascending powers of x, up to and including x^2.
State the range of x for which the expansion is valid.

$(16 + x)^{\frac{1}{2}} = 16^{\frac{1}{2}}(1 + \frac{1x}{16})^{\frac{1}{2}}$

$16^{\frac{1}{2}} = 4$

Expand $(1 + y)^{\frac{1}{2}}$ with $y = \frac{1x}{16}$

Valid for $|\frac{1x}{16}| < 1$, i.e. $|x| < 16$

Answer: $4 + \frac{1}{8}x - \frac{1}{512}x^2$; $|x| < 16$

9 Implicit differentiation: tangent to a curve (All boards)

A curve has equation $-2x^2 + xy + 3x = -21$.
Show that P(−3, −2) lies on the curve and find the equation of the tangent at P in the form $ax + by + c = 0$ with integer a, b, c.

Differentiate (product rule on the xy term): $(-4x + y + 3) + (x)\frac{dy}{dx} = 0$

At P(−3, −2): $13 + (-3)\frac{dy}{dx} = 0$, so $\frac{dy}{dx} = \frac{13}{3}$

Tangent: $y + 2 = \frac{13}{3}(x + 3)$, i.e. $13x - 3y + 33 = 0$

Answer: $13x - 3y + 33 = 0$

10 Acceleration function of time (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle moves on the x-axis. At time t s its acceleration is $(3t - 6)$ m s^{-2}. When $t = 0$ it is at O with velocity -6 m s^{-1}.
Find (a) its velocity when the acceleration is zero, (b) its displacement from O then.

$a = 0$ when $t = 2$
$v = \int a\,dt = 1.5t^2 - 6t - 6$ (using $v = -6$ when $t = 0$)
$v(2) = -12$
$x = \int v\,dt = 0.5t^3 - 3t^2 - 6t$ ($x = 0$ when $t = 0$)
$x(2) = -20$
Answer: (a) -12 m s^{-1} (b) -20 m