Day 99 **S Tier 10-a-day: Model Solutions**

1 Prime factors and indices

Write $10^5 \times 14^4$ as a product of powers of prime numbers.

$10 = 2 \times 5$, $14 = 2 \times 7$

$10^5 = 2^5 \times 5^5$

$14^4 = 2^4 \times 7^4$

Add the powers of each prime: $2^9 \times 5^5 \times 7^4$

Answer: $2^9 \times 5^5 \times 7^4$

2 Estimated mean with an unknown frequency

The table gives information about the times taken by some people to travel to work.
An estimate for the mean is 23.
Work out the value of x.

Mid-points: 5, 15, 25, 35

Estimated mean = $\Sigma fm \div \Sigma f$, so $(870 + 15x) \div (34 + x) = 23$

$870 + 15x = 23(34 + x) = 782 + 23x$

$(15 - 23)x = 782 - 870$

$-8x = -88$, so $x = 11$

Answer: x = 11

3 Alternate segment theorem

PAQ is a tangent to the circle at A. B and C are points on the circle. Angle PAB = 46° and angle CAQ = 69°.
(a) Work out the size of angle ABC.
(b) Work out the size of angle ACB.
The diagram is not drawn accurately.

(a) Angle ABC = angle CAQ = 69° (alternate segment theorem)

(b) Angle ACB = angle PAB = 46° (alternate segment theorem)

Answer: (a) 69° (b) 46°

4 Average speed

A car travels the first half of a journey at 30 km/h.
It travels the second half of the same journey at 60 km/h.
Work out the average speed for the whole journey.

Let each half of the journey be 60 km (a common multiple of 30 and 60)

Time for first half = 60 ÷ 30 = 2 hours

Time for second half = 60 ÷ 60 = 1 hours

Average speed = total distance ÷ total time = 120 ÷ 3 = 40 km/h

Answer: 40 km/h

5 Trigonometric equations

Solve $2 \tan x° - 3 = -1$ for $0 \leq x \leq 360$

Rearrange: $2 \tan x° = 2$, so $\tan x° = 1$

The acute angle with tangent 1 is 45°; tangent is positive in the first and third quadrants

x = 45 or x = 225 (45 and 180 + 45)

Answer: x = 45, 225

6 Index equations

Solve $3^{2x+2} = 27^4$

Write both sides as powers of 3: $27 = 3^3$

Equate the powers: $2x + 2 = 12$

$2x = 10$, so $x = 5$

Answer: x = 5

Day 99 **S Tier 10-a-day: Model Solutions**

7 3D problems: pyramids

VABCD is a pyramid on a square base. V is directly above the centre M.
(a) Work out the height VM of the pyramid.
(b) Work out the volume of the pyramid.
The diagram is not drawn accurately.

(a) $AC^2 = 8^2 + 8^2 = 128$, so $AM^2 = (AC \div 2)^2 = 32$
$VM^2 = VA^2 - AM^2 = 9^2 - 32 = 49$
VM = 7 cm

(b) Volume = $\frac{1}{3} \times 8^2 \times 7 = \frac{448}{3}$ cm^3

Answer: (a) 7 cm (b) $\frac{448}{3}$ cm^3

8 Cosine rule and area of a triangle

The diagram shows triangle ABC. (Calculator)
(a) Work out the length of AB, marked x, to 1 decimal place.
(b) Work out the area of triangle ABC to 3 significant figures.
The diagram is not drawn accurately.

(a) $x^2 = 5^2 + 10^2 - 2 \times 5 \times 10 \times \cos 70°$
$x^2 = 90.798$
x = 9.5 cm (1 d.p.)

(b) Area = $\frac{1}{2} \times 5 \times 10 \times \sin 70° = 23.5$ cm^2 (3 s.f.)

Answer: (a) 9.5 cm (b) 23.5 cm^2

9 Nested surds

Show that $\sqrt{43 + 30\sqrt{2}} = 5 + 3\sqrt{2}$

Square the right-hand side: $(5 + 3\sqrt{2})^2 = 25 + 30\sqrt{2} + 18$
$= 43 + 30\sqrt{2}$
So $\sqrt{43 + 30\sqrt{2}} = 5 + 3\sqrt{2}$

Answer: $\sqrt{43 + 30\sqrt{2}} = 5 + 3\sqrt{2}$

10 Algebraic fractions: adding and subtracting

Write $\frac{7}{x-2} - \frac{x+3}{x+4}$ as a single fraction.
Simplify the numerator.

The common denominator is $(x - 2)(x + 4)$
$$\frac{7}{x-2} - \frac{x+3}{x+4} = \frac{7(x+4) - (x+3)(x-2)}{(x-2)(x+4)}$$
Expand the numerator: $-x^2 + 6x + 34$
$$\frac{-x^2 + 6x + 34}{(x-2)(x+4)}$$

Answer: $\frac{-x^2 + 6x + 34}{(x-2)(x+4)}$