Day 97 **S Tier 10-a-day: Model Solutions**

1 Substitution

Work out the value of $\frac{2p - q}{r}$

when p = 5, q = −2 and r = 1.

Substitute the values: $\frac{2 \times 5 - (-2)}{1}$

$= \frac{12}{1}$

$= 12$

Answer: 12

2 Angles with algebra

In triangle PQR, side QR is extended to S.
Angle QPR = (4x − 15)°
Angle PQR = (x + 25)°
Angle PRS = (3x + 56)°
(a) Work out the value of x.
(b) Work out the size of angle PRQ.

(a) The exterior angle of a triangle equals the sum of the two opposite interior angles
3x + 56 = 5x + 10
2x = 46
x = 23
(b) Angle PRS = 125° so angle PRQ = 180° − 125° = 55°
Answer: (a) x = 23 (b) 55°

3 Quadratic graphs

The curve $y = x^2 - 6x + 8$ crosses the x-axis at the points A and B.
(a) Find the coordinates of A and B.
(b) Find the coordinates of the minimum point of the curve.

(a) Put y = 0: $x^2 - 6x + 8 = 0$, so $(x - 2)(x - 4) = 0$
x − 2 = 0 or x − 4 = 0, so x = 2 or x = 4
(b) The turning point is halfway between A and B: x = (2 + 4) ÷ 2 = 3
When x = 3: $y = 3^2 - 6 \times 3 + 8 = -1$
Answer: (a) A(2, 0), B(4, 0) (b) (3, −1)

4 Rationalising the denominator

Rationalise the denominator of $\frac{22}{4\sqrt{3}}$

Give your answer in its simplest form.

Multiply top and bottom by $\sqrt{3}$:

$\frac{22}{4\sqrt{3}} = \frac{22\sqrt{3}}{12}$

$= \frac{11\sqrt{3}}{6}$

Answer: $\frac{11\sqrt{3}}{6}$

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5 Arc length and sector area

A sector of a circle has radius 6 cm.

The length of the arc of the sector is $\frac{9}{2}\pi$ cm.

(a) Work out the angle of the sector.
(b) Work out the area of the sector. Give your answer to (b) in terms of π.

(a) Arc length = $\frac{\theta}{360} \times 2\pi \times 6$, so $\frac{\theta}{360} \times 12 = \frac{9}{2}$

$\theta = \frac{9}{2} \times \frac{360}{12} = 135°$

(b) Area = $\frac{135}{360} \times \pi \times 6^2 = \frac{27}{2}\pi$ cm^2

Answer: (a) 135° (b) $\frac{27}{2}\pi$ cm^2

6 Line of best fit

The scatter graph shows hours of revision (x) and test mark (y).
The line of best fit passes through (2, 15) and (8, 27).
(a) Find its equation in the form y = mx + c.
(b) Explain what its gradient tells you.

(a) Gradient m = $\frac{27 - 15}{8 - 2} = \frac{12}{6} = 2$

y = 2x + c: using (2, 15), 15 = 2 × 2 + c, so c = 11

y = 2x + 11

(b) For each extra hour of revision, the test mark goes up by 2

Answer: (a) y = 2x + 11 (b) For each extra hour of revision, the test mark goes up by 2

7 Angles of elevation

A point A is on level ground in line with the base of a cliff. (Calculator)
The angle of elevation of the top of the cliff from A is 55°.
B is a point on the same line, 40 m further from the cliff than A.
The angle of elevation of the top of the cliff from B is 21°.
Work out the height of the cliff. Give your answer to 1 decimal place.

Let the height be h and the distance from A to the base be x. tan 55° = $\frac{h}{x}$, so x = h ÷ tan 55°

From B: tan 21° = $\frac{h}{x + 40}$, so x + 40 = h ÷ tan 21°

Subtract: 40 = h ÷ tan 21° − h ÷ tan 55° = h(2.6051 − 0.70021)

h = 40 ÷ 1.9049 = 21.0 m (1 d.p.)

Answer: 21.0 m

8 Algebraic fractions: multiplying and dividing

Simplify fully $\frac{x^2 - x - 2}{x^2 - 7x} \times \frac{x^2 + 4x}{x + 1}$

Factorise: $\frac{(x + 1)(x - 2)}{x(x - 7)} \times \frac{x(x + 4)}{(x + 1)}$

Cancel (x + 1) and x

= $\frac{(x - 2)(x + 4)}{(x - 7)}$

Answer: $\frac{(x - 2)(x + 4)}{(x - 7)}$

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9 Counting factors

$N = 2^4 \times 3 \times 5^3$

How many of the factors of N are even numbers?

An even factor must contain at least one 2
Power of 2: 1 to 4, that is 4 choices
Power of 3: 0 to 1, that is 2 choices
Power of 5: 0 to 3, that is 4 choices
Total = 4 × 2 × 4 = 32
Answer: 32

10 Exponential growth and decay

A radioactive sample has a mass of 480 g. Its mass halves every 8 hours.
(a) Work out the mass after 16 hours.
(b) After how many hours will the mass first be less than 5 g?

(a) 16 hours is 2 half-lives: $480 \div 2^2 = 120$ g
(b) Halve repeatedly: 480, 240, 120, 60, 30, 15, 7.5, 3.75
3.75 is the first value below 5, after 7 half-lives, so 7 × 8 = 56 hours
Answer: (a) 120 g (b) 56 hours