Day 96 **S Tier 10-a-day: Model Solutions**

1 Area of a triangle using sine

In triangle PQR, PQ = 4 cm and angle PQR = 30°.
The area of the triangle is 11 cm^2.
Work out the length of QR.
Do not use a calculator.

Area = $\frac{1}{2} \times PQ \times QR \times \sin Q$

$11 = \frac{1}{2} \times 4 \times QR \times \sin 30°$

$\sin 30° = \frac{1}{2}$, so $11 = 1 \times QR$

QR = 11 ÷ 1 = 11 cm

Answer: 11 cm

2 Inequalities

(a) Solve $-4 - 3x < 5$
(b) n is an integer. Write down all the values of n that satisfy
$-2 < 3n + 1 \leq 13$

(a) Add 4: $-3x < 9$
Divide by -3 and reverse the inequality sign: $x > -3$
(b) Subtract 1: $-3 < 3n \leq 12$
Divide by 3: $-1 < n \leq 4$
Integers: 0, 1, 2, 3, 4
Answer: (a) $x > -3$ (b) 0, 1, 2, 3, 4

3 Combined transformations

The point P(4, −2) is reflected in the y-axis.
The image is then rotated 90° clockwise about the origin.
(a) Write down the coordinates of the final image of P.
(b) Describe fully the single transformation that is equivalent to these two transformations.

(a) After the first transformation: P' = (−4, −2)
After the second transformation: P'' = (−2, 4)
(b) Follow a general point (x, y) through both transformations: (x, y) → (−x, y) → (y, x)
The single mapping is (x, y) → (y, x)
This is a reflection in the line y = x
Answer: (a) (−2, 4) (b) Reflection in the line y = x

4 Growth and decay

A machine is worth £8,000 when new.
Its value decreases by 5% each year.
After how many whole years will it first be worth less than £5,350? (Calculator)

Multiplier each year = 0.95
After 7 years: £8,000 × 0.95^7 = £5,586.70
After 8 years: £8,000 × 0.95^8 = £5,307.36
So the first time it is less than £5,350 is after 8 years
Answer: 8 years

Day 96 **S Tier 10-a-day: Model Solutions**

5 Perpendicular bisector

A is the point $(-2, 1)$ and B is the point $(2, -5)$.
Find an equation of the perpendicular bisector of AB. Give your answer in the form $ax + by = c$, where a, b and c are integers.

Midpoint of AB = $(0, -2)$

Gradient of AB = $-\frac{3}{2}$

Perpendicular gradient = $\frac{2}{3}$

$y - (-2) = \frac{2}{3}(x - 0)$

Rearrange to integer coefficients: $2x - 3y = 6$

Answer: $2x - 3y = 6$

6 Simplifying algebraic fractions

Simplify fully $\frac{x^2 - 4x - 21}{x^2 + 7x + 12}$

Factorise the top: $(x + 3)(x - 7)$
Factorise the bottom: $(x + 3)(x + 4)$

Cancel the common factor $(x + 3)$: $\frac{x - 7}{x + 4}$

Answer: $\frac{x - 7}{x + 4}$

7 Index equations

Solve $3^{3x + 3} = 27^{2x + 3}$

Write both sides as powers of 3: $27 = 3^3$
Equate the powers: $3x + 3 = 6x + 9$
$-3x = 6$, so $x = -2$
Answer: $x = -2$

8 Estimating the median from grouped data

The table gives information about 84 lengths.
Work out an estimate for the median.
Assume the values in each class are evenly spread.

Cumulative frequencies: 9, 27, 31, 35, 60, 84
The median is the value at position $84 \div 2 = 42$
It is in the class 100 to 120, with 35 values before it, so we need 7 more of the 25 values in this class

Median $\approx 100 + \frac{7}{25} \times 20 = 105.6$

Answer: 105.6

9 LCM problems

A church bell rings every 5 minutes.
Another bell rings every 10 minutes.
A third bell rings every 14 minutes.
They all ring together at 7:00 am.
How many times do they all ring together between 7:00 am and 3:00 pm, including both 7:00 am and 3:00 pm if they do?

Find the LCM of 5, 10 and 14
$5 = 5$, $10 = 2 \times 5$, $14 = 2 \times 7$
LCM = $2 \times 5 \times 7 = 70$
Total time from 7:00 am to 3:00 pm = 480 minutes
$480 \div 70 = 6.86$, so there are 6 more times after the start
Including the start: $6 + 1 = 7$
Answer: 7

Day 96 **S Tier 10-a-day: Model Solutions**

10 Estimated mean with an unknown frequency

The table gives information about the masses of some eggs.
An estimate for the mean is 22.5.
Work out the value of x.

Mid-points: 12.5, 17.5, 22.5, 27.5, 32.5
Estimated mean = $\Sigma fm \div \Sigma f$, so $(847.5 + 17.5x) \div (35 + x) = 22.5$
$847.5 + 17.5x = 22.5(35 + x) = 787.5 + 22.5x$
$(17.5 - 22.5)x = 787.5 - 847.5$
$-5x = -60$, so $x = 12$
Answer: x = 12