Day 95 **S Tier 10-a-day: Model Solutions**

1 Circle theorems: angle at the centre

A, B and C are points on a circle, centre O. Angle OAB = 50°.
(a) Work out the size of angle AOB.
(b) Work out the size of angle ACB.
The diagram is not drawn accurately.

(a) OA = OB (radii), so triangle OAB is isosceles and angle OBA = 50°
Angle AOB = 180 − 2 × 50 = 80°
(b) The angle at the circumference is half the angle at the centre: 80 ÷ 2 = 40°
Answer: (a) 80° (b) 40°

2 Capture and recapture

100 fish in a lake are caught, tagged and put back.
Later, a second sample of 120 fish is caught. 4 of these are tagged.
(a) Work out an estimate for the number of fish in a lake.
(b) State one assumption you have made.

(a) The proportion tagged in the second sample is 4 out of 120
This should match the proportion tagged in the whole population: $\frac{4}{120} = \frac{100}{N}$
N = 100 × 120 ÷ 4 = 3000
(b) No fish are born, die or leave between the samples, and the fish mix evenly
Answer: (a) 3000 (b) The population stays the same between samples

3 Expanding three brackets

Expand and simplify (x − 5)(x + 2)(x + 6)

Expand the first two brackets: $(x - 5)(x + 2) = x^2 - 3x - 10$
Multiply by (x + 6): $x(x^2 - 3x - 10) + 6(x^2 - 3x - 10)$
$= x^3 + 3x^2 - 28x - 60$
Answer: $x^3 + 3x^2 - 28x - 60$

4 Transformations of graphs

The graph of y = f(x) has a maximum point at (3, −5).
Write down the coordinates of the turning point of the graph of
(a) y = f(x) + 3
(b) y = f(x + 1) − 2

(a) Start with the turning point (3, −5)
Apply the transformation to the point: (3, −2)
(b) f(x + 1) changes the x-coordinate to 2, and f(x) − 2 changes the y-coordinate to −7
Turning point: (2, −7)
Answer: (a) (3, −2) (b) (2, −7)

5 Rational and irrational numbers

Here are six numbers.
$\frac{1}{\sqrt{3}}, \sqrt{2} \times \sqrt{6}, \frac{\sqrt{18}}{\sqrt{2}}, \frac{\sqrt{20}}{\sqrt{5}}, (1 + \sqrt{2})^2, \sqrt{2} + \sqrt{8}$
Write down the rational numbers.

Simplify each number:
$\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$; $\sqrt{2} \times \sqrt{6} = \sqrt{12} = 2\sqrt{3}$; $\frac{\sqrt{18}}{\sqrt{2}} = \sqrt{9} = 3$
$\frac{\sqrt{20}}{\sqrt{5}} = \sqrt{4} = 2$; $(1 + \sqrt{2})^2 = 3 + 2\sqrt{2}$; $\sqrt{2} + \sqrt{8} = 3\sqrt{2}$
Answer: $\frac{\sqrt{18}}{\sqrt{2}}, \frac{\sqrt{20}}{\sqrt{5}}$

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6 Trigonometric graphs

The diagram shows the graph of $y = a \cos x° + c$ for $0 \leq x \leq 360$.
Some points on the graph are labelled.
Find the value of a and the value of c.

At x = 0, cos x° = 1, so a + c = 11
At x = 180, cos x° = −1, so −a + c = 3
Add: 2c = 14, so c = 7
Subtract: 2a = 8, so a = 4
Answer: a = 4, c = 7

7 Reverse percentage growth and decay

The value of a house increases by 20% each year.
After 4 years it is worth £40,435.20.
Work out its value 4 years ago. (Calculator)

Yearly multiplier = 1.2
Original × 1.2^4 = £40,435.20
Original = £40,435.20 ÷ 1.2^4 = £19,500
Answer: £19,500

8 Angles of elevation

A point A is on level ground in line with the base of a cliff. (Calculator)
The angle of elevation of the top of the cliff from A is 46°.
B is a point on the same line, 60 m further from the cliff than A.
The angle of elevation of the top of the cliff from B is 37°.
Work out the height of the cliff. Give your answer to 1 decimal place.

Let the height be h and the distance from A to the base be x. $\tan 46° = \frac{h}{x}$, so $x = h \div \tan 46°$

From B: $\tan 37° = \frac{h}{x + 60}$, so $x + 60 = h \div \tan 37°$

Subtract: $60 = h \div \tan 37° - h \div \tan 46° = h(1.3270 - 0.96569)$
h = 60 ÷ 0.36136 = 166.0 m (1 d.p.)
Answer: 166.0 m

9 Mixtures and percentage profit

A shop mixes two types of mixed nuts.
Type A costs £9.00 per kg and type B costs £12.00 per kg.
They are mixed in the ratio A : B = 2 : 4.
The mixture is sold for £13.20 per kg.
Work out the percentage profit.

Cost of 6 kg of mixture = 2 × £9.00 + 4 × £12.00 = £66.00
Cost per kg = £66.00 ÷ 6 = £11.00
Profit per kg = £13.20 − £11.00 = £2.20
Percentage profit = 2.20 ÷ 11.00 × 100 = 20%
Answer: 20%

10 Proof with indices

Show that $3^{n+3} + 3^n$ is always a multiple of 28.

$3^{n+3} = 3^n \times 3^3$
$3^{n+3} + 3^n = 3^n(3^3 + 1)$
$3^3 + 1 = 28$
$= 28 \times 3^n$, which is a multiple of 28
Answer: 28×3^n