Day 94 **S Tier 10-a-day: Model Solutions**

1 Substitution

Work out the value of $3p - q^3$
when p = 1, q = 1 and r = 2.

Substitute the values: $3 \times 1 - 1^3$
$= 3 - 1$
$= 2$
Answer: 2

2 Weighted percentages

The ratio of men to women in a school is 5 : 1.
80% of the men and 20% of the women own a car.
What percentage of all of these students own a car?

Imagine 120 students in total: 100 men and 20 women
80% of 100 = 80 and 20% of 20 = 4
Together: 84 out of 120
84 ÷ 120 × 100 = 70%
Answer: 70%

3 Circle theorems: angle at the centre

A, B and C are points on a circle, centre O. Angle ACB = 32°.
(a) Work out the size of angle AOB.
(b) Work out the size of angle OAB.
The diagram is not drawn accurately.

(a) The angle at the centre is twice the angle at the circumference: 2 × 32 = 64°
(b) OA = OB (radii), so triangle OAB is isosceles
Angle OAB = (180 − 64) ÷ 2 = 58°
Answer: (a) 64° (b) 58°

4 Cumulative frequency graphs

The cumulative frequency graph gives information about the masses of 120 apples picked from a tree.
(a) Find an estimate for the median and the interquartile range.
(b) Estimate how many of the 120 had a mass greater than 15.

(a) Median at cumulative frequency 60, so median = 30
Lower quartile at 30, so LQ = 20. Upper quartile at 90, so UQ = 40
Interquartile range = 40 − 20 = 20
(b) At 15, cumulative frequency = 24
Number greater = 120 − 24 = 96
Answer: (a) median 30, IQR 20 (b) 96

5 Fraction problems

A bag contains counters.
$\frac{1}{6}$ of the counters are red and $\frac{3}{5}$ of the counters are blue.
The rest of the counters, 14 of them, are green.
How many red counters are there?

Fraction that are green = $1 - \frac{1}{6} - \frac{3}{5} = \frac{7}{30}$

$\frac{7}{30}$ of the total is 14

Total = $14 \div \frac{7}{30} = 60$

Red counters = $\frac{1}{6}$ of 60 = 10

Answer: 10

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6 Laws of indices with algebra

Simplify $(81x^4y^{-4})^{\frac{1}{2}}$

Raise each part to the power $\frac{1}{2}$

$81^{\frac{1}{2}} = \sqrt{81} = 9$

$(x^4)^{\frac{1}{2}} = x^2$ and $(y^{-4})^{\frac{1}{2}} = y^{-2}$

$= 9x^2y^{-2}$

Answer: $9x^2y^{-2}$

7 Exact trigonometric ratios from a given ratio

x is an obtuse angle and $\sin x = \frac{1}{7}$.

(a) Find the exact value of cos x.

(b) Find the exact value of tan x. Give each answer as a fraction or a surd. Do not use a calculator.

(a) $\cos^2 x = 1 - (\frac{1}{7})^2 = \frac{48}{49}$

x is obtuse, so cos x is negative: $\cos x = -\frac{4\sqrt{3}}{7}$

(b) $\tan x = \frac{\sin x}{\cos x} = \frac{1}{7} \div (-\frac{4\sqrt{3}}{7}) = -\frac{\sqrt{3}}{12}$

Answer: (a) $-\frac{4\sqrt{3}}{7}$ (b) $-\frac{\sqrt{3}}{12}$

8 Counting factors

How many factors does 700 have?

Include 1 and 700.

$700 = 2^2 \times 5^2 \times 7$

Add 1 to each power and multiply: $3 \times 3 \times 2 = 18$

Answer: 18

9 Tangency conditions

The line $y = -x + k$ is a tangent to the curve $y = 3x^2 + 4x + 1$.

Find the value of k.

Equate: $3x^2 + 4x + 1 = -x + k$

Rearrange: $3x^2 + 5x + 1 - k = 0$

A tangent touches at one point, so $b^2 - 4ac = 0$: $25 - 4 \times 3 \times (1 - k) = 0$

$12k + 13 = 0$

$k = -\frac{13}{12}$

Answer: $k = -\frac{13}{12}$

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10 Vectors: intersecting lines

OAB is a triangle. OA = $\mathbf{a}$ and OB = $\mathbf{b}$.
M is on OA with OM : MA = 3 : 1. N is on OB with ON : NB = 1 : 1. AN and BM intersect at X.
(a) Find BM in terms of $\mathbf{a}$ and $\mathbf{b}$.
(b) Find OX in terms of $\mathbf{a}$ and $\mathbf{b}$.
The diagram is not drawn accurately.

(a) OM = $\frac{3}{4}\mathbf{a}$, so BM = $-\mathbf{b} + \frac{3}{4}\mathbf{a} = \frac{3}{4}\mathbf{a} - \mathbf{b}$

(b) X is on BM: BX = s BM, so OX = OB + s BM = $\frac{3}{4}s\mathbf{a} + (1 - s)\mathbf{b}$

X is on AN: OX = OA + t AN = OA + t(ON − OA) = $(1 - t)\mathbf{a} + \frac{1}{2}t\mathbf{b}$

Equate: $\frac{3}{4}s = 1 - t$ and $1 - s = \frac{1}{2}t$

Solving: $s = \frac{4}{5}$ and $t = \frac{2}{5}$

OX = $\frac{3}{5}\mathbf{a} + \frac{1}{5}\mathbf{b}$

Answer: (a) $\frac{3}{4}\mathbf{a} - \mathbf{b}$ (b) OX = $\frac{3}{5}\mathbf{a} + \frac{1}{5}\mathbf{b}$