Day 93 **S Tier 10-a-day: Model Solutions**

1 Capture and recapture

80 deer in a forest are caught, marked and put back.
Later, a second sample of 50 deer is caught. 2 of these are marked.
(a) Work out an estimate for the number of deer in a forest.
(b) State one assumption you have made.

(a) The proportion marked in the second sample is 2 out of 50

This should match the proportion marked in the whole population: $\frac{2}{50} = \frac{80}{N}$

$N = 80 \times 50 \div 2 = 2000$

(b) No deer are born, die or leave between the samples, and the deer mix evenly

Answer: (a) 2000 (b) The population stays the same between samples

2 Solving quadratics by factorising

Solve $9x^2 - 8 = 6x$

Rearrange so that one side is 0: $9x^2 - 6x - 8 = 0$
Factorise: $(3x + 2)(3x - 4) = 0$
$3x + 2 = 0$ or $3x - 4 = 0$
$x = -\frac{2}{3}$ or $x = \frac{4}{3}$

Answer: $x = -\frac{2}{3}$ or $x = \frac{4}{3}$

3 Growth and decay

A culture has 5000 bacteria.
The number of bacteria increases by 8% each hour.
After how many whole hours will there first be more than 12,500 bacteria? (Calculator)

Multiplier each hour = 1.08

After 11 hours: $5000 \times 1.08^{11} = 11{,}658$

After 12 hours: $5000 \times 1.08^{12} = 12{,}591$

So the first time it is more than 12,500 is after 12 hours

Answer: 12 hours

4 Perimeter and area of a running track

The diagram shows a running track made from a rectangle and two semicircles. (Calculator)
An athlete runs 5 complete laps around the track.
Work out the total distance run, to the nearest 10 metres.
The diagram is not drawn accurately.

Boundary of one lap = $2 \times 40 + 2 \times \pi \times 20 = 205.66$ m
5 laps = $5 \times 205.66 = 1028.3$ m
= 1030 m to the nearest 10 m

Answer: 1030 m

5 Laws of indices

Given that $\frac{5^3 \times 25^3}{25^2} = 5^n$,
find the value of n.

$25 = 5^2$

Numerator: $5^3 \times 5^6 = 5^9$

Denominator: 5^4

Subtract the powers: $9 - 4 = 5$

Answer: $n = 5$

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6 Gradient of a chord

A and B are points on the curve $y = 3x^2 + x + 4$.
The x coordinate of A is 0 and the x coordinate of B is 2.
Work out the gradient of the line AB.

At A: y = 4. At B: y = 18
A is (0, 4) and B is (2, 18)
Gradient = $\frac{18 - 4}{2 - 0} = \frac{14}{2} = 7$

Answer: 7

7 Arc length and sector area

A sector of a circle has radius 15 cm and angle 120°.
(a) Work out the length of the arc of the sector.
(b) Work out the area of the sector. Give each answer in terms of π.

(a) Fraction of the circle = $\frac{120}{360}$

Arc length = $\frac{120}{360} \times 2 \times \pi \times 15 = 10\pi$ cm

(b) Area = $\frac{120}{360} \times \pi \times 15^2 = 75\pi$ cm^2

Answer: (a) 10π cm (b) 75π cm^2

8 Mixtures and percentage profit

A shop mixes two types of tea.
Type A costs £4.00 per kg and type B costs £13.00 per kg.
They are mixed in the ratio A : B = 1 : 4.
The mixture is sold for £20.16 per kg.
Work out the percentage profit.

Cost of 5 kg of mixture = 1 × £4.00 + 4 × £13.00 = £56.00
Cost per kg = £56.00 ÷ 5 = £11.20
Profit per kg = £20.16 − £11.20 = £8.96
Percentage profit = 8.96 ÷ 11.20 × 100 = 80%
Answer: 80%

9 Vectors: intersecting lines

OAB is a triangle. OA = **a** and OB = **b**.
M is on OA with OM : MA = 3 : 1. N is on OB with ON : NB = 1 : 3. AN and BM intersect at X.
(a) Find BM in terms of **a** and **b**.
(b) Find OX in terms of **a** and **b**.
The diagram is not drawn accurately.

(a) OM = $\frac{3}{4}\mathbf{a}$, so BM = $-\mathbf{b} + \frac{3}{4}\mathbf{a} = \frac{3}{4}\mathbf{a} - \mathbf{b}$

(b) X is on BM: BX = s BM, so OX = OB + s BM = $\frac{3}{4}s\mathbf{a} + (1 - s)\mathbf{b}$

X is on AN: OX = OA + t AN = OA + t(ON − OA) = $(1 - t)\mathbf{a} + \frac{1}{4}t\mathbf{b}$

Equate: $\frac{3}{4}s = 1 - t$ and $1 - s = \frac{1}{4}t$

Solving: $s = \frac{12}{13}$ and $t = \frac{4}{13}$

OX = $\frac{9}{13}\mathbf{a} + \frac{1}{13}\mathbf{b}$

Answer: (a) $\frac{3}{4}\mathbf{a} - \mathbf{b}$ (b) OX = $\frac{9}{13}\mathbf{a} + \frac{1}{13}\mathbf{b}$

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10 Bounds and accuracy

a = 20.5 and b = 0.692, both correct to 3 significant figures.
Work out a ÷ b.
Write down your answer to a suitable degree of accuracy.
You must show your working and give a reason for your answer. (Calculator)

$20.45 \leq a < 20.55$; $0.6915 \leq b < 0.6925$
Lower bound = 20.45 ÷ 0.6925 = 29.531...
Upper bound = 20.55 ÷ 0.6915 = 29.718...
Both bounds round to 30 to 2 significant figures, but not to 3 significant figures
Answer: 30 (2 s.f.)