Day 92 **S Tier 10-a-day: Model Solutions**

1 Mean of a changing set

The mean of 9 numbers is 89.
A new number is added and the mean of all 10 numbers is 80.7.
Work out the new number.

Total of the 9 numbers = 9 × 89 = 801
New total = 10 × 80.7 = 807
New number = 807 − 801 = 6
Answer: 6

2 Area of a triangle using sine

In triangle PQR, PQ = 12 cm and angle PQR = 45°.
The area of the triangle is $60\sqrt{2}$ cm^2.
Work out the length of QR.
Do not use a calculator.

Area = $\frac{1}{2}$ × PQ × QR × sin Q

$60\sqrt{2} = \frac{1}{2} \times 12 \times QR \times \sin 45°$

$\sin 45° = \frac{\sqrt{2}}{2}$, so $60\sqrt{2} = 3\sqrt{2} \times QR$

QR = $60\sqrt{2} \div 3\sqrt{2}$ = 20 cm
Answer: 20 cm

3 Solving quadratics by factorising

Solve $6x^2 - 25x + 14 = 0$

Factorise: (3x − 2)(2x − 7) = 0
3x − 2 = 0 or 2x − 7 = 0
$x = \frac{2}{3}$ or $x = \frac{7}{2}$
Answer: $x = \frac{2}{3}$ or $x = \frac{7}{2}$

4 Averages with algebra

Here are five numbers: 1, 3x − 3, 2x + 5, 2x − 1, x − 3.
The mean of the five numbers is 11.
(a) Work out the value of x.
(b) Work out the range of the five numbers.

(a) Total = (1) + (3x − 3) + (2x + 5) + (2x − 1) + (x − 3) = 11 × 5 = 55
8x − 1 = 55, so 8x = 56 and x = 7
The numbers are 1, 18, 19, 13, 4
(b) Range = 19 − 1 = 18
Answer: (a) x = 7 (b) 18

5 Standard form

Work out $(6 \times 10^8) \div (1.5 \times 10^{-7})$
Give your answer in standard form.

6 ÷ 1.5 = 4
$10^8 \div 10^{-7} = 10^{15}$
$= 4 \times 10^{15}$
Answer: 4×10^{15}

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6 Exact trigonometry in triangles

ABC is a right-angled triangle.
Work out the exact value of x. Give your answer in its simplest form.
The diagram is not drawn accurately.

x is the hypotenuse AC and the known side is the opposite side BC, so use sin 60°

$\sin 60° = \frac{\sqrt{3}}{2}$

$x = 5 \div \sin 60° = 5 \div \frac{\sqrt{3}}{2}$

$x = \frac{10\sqrt{3}}{3}$ (after rationalising the denominator)

Answer: x = $\frac{10\sqrt{3}}{3}$

7 Pythagoras and quadratic equations

The lengths of the sides of a right-angled triangle are (x + 1) cm, (x − 6) cm and (x + 2) cm.
The longest side is (x + 2) cm.
(a) Show that $x^2 - 14x + 33 = 0$
(b) Hence work out the area of the triangle.

(a) Pythagoras: $(x + 1)^2 + (x - 6)^2 = (x + 2)^2$
Expand: $x^2 + 2x + 1 + x^2 - 12x + 36 = x^2 + 4x + 4$
Rearrange: $x^2 - 14x + 33 = 0$
(b) Factorise: $(x - 11)(x - 3) = 0$, so x = 11 (the other root, x = 3, does not give valid side lengths)
The shorter sides are 12 cm and 5 cm, so area = $\frac{1}{2} \times 12 \times 5 = 30$ cm^2

Answer: (b) 30 cm^2

8 Laws of indices

Write $\frac{9^n \times 3^{n-1}}{9^n}$ in the form 3^{an+c}
where a and c are integers.

$9 = 3^2$, $3 = 3^1$, $9 = 3^2$
$9^n = 3^{2(n)}$, $3^{n-1} = 3^{n-1}$, $9^n = 3^{2n}$
Exponent: $2(n) + (n - 1) - 2n = n - 1$
Answer: a = 1, c = −1

9 Ratio and expressions

x : y = 2 : 9
Work out the ratio (5x + 1y) : (3y − 3x)
Give your answer in its simplest form.

x : y = 2 : 9, so let x = 2t and y = 9t
5(2t) + 1(9t) = 19t
3(9t) − 3(2t) = 21t
19t : 21t = 19 : 21
Answer: 19 : 21

10 Bounds in geometry

A rectangle has length 7.0 cm and width 4.8 cm, both correct to 1 decimal place.
Work out the upper bound of the perimeter of the rectangle.

Each length is given to 1 decimal place, so it can be up to 0.05 cm either side
Length is between 6.95 and 7.05, width is between 4.75 and 4.85
Perimeter = 2 × (7.05 + 4.85) = 23.8 cm
Answer: 23.8 cm