Day 91 **S Tier 10-a-day: Model Solutions**

1 Substitution

Work out the value of $x^2 - 3yz$
when $x = 3$, $y = -4$ and $z = 2$.

Substitute the values: $3^2 - 3 \times (-4) \times 2$
$= 9 - (-24)$
$= 33$
Answer: 33

2 Inverse proportion in context

6 painters can paint a hall in 15 days.
After 4 days, 3 of the painters leave. The rest carry on working at the same rate.
How many more days does it take to finish?

Total work = $6 \times 15 = 90$ painter-days
Work done = $6 \times 4 = 24$, so work left = 66
Workers left = $6 - 3 = 3$
Time = $66 \div 3 = 22$ days
Answer: 22 days

3 Chords and Pythagoras in circles

AB is a chord of a circle, centre O. M is the midpoint of AB. (Calculator)
(a) Work out the radius of the circle.
(b) Work out angle AOB, to 1 decimal place.
The diagram is not drawn accurately.

(a) AM = $10 \div 2 = 5$ cm
$OA^2 = OM^2 + AM^2 = 12^2 + 5^2 = 169$
OA = 13 cm

(b) sin(angle AOM) = $\frac{5}{13}$

Angle AOM = 22.62°, so angle AOB = $2 \times 22.62 = 45.2°$
Answer: (a) 13 cm (b) 45.2°

4 Distance-time graphs

The graph shows Noah's cycle ride to a café and back.
(a) Work out Noah's speed on the way back.
(b) Work out Noah's average speed for the whole ride, including the stop.

(a) Return: 24 km in 1 hours, so speed = $24 \div 1 = 24$ km/h
(b) Total distance = $2 \times 24 = 48$ km, total time = 3 hours
Average speed = $48 \div 3 = 16$ km/h
Answer: (a) 24 km/h (b) 16 km/h

5 Arc length and sector area

A sector of a circle has radius 7 cm.

The length of the arc of the sector is $\frac{7}{2}\pi$ cm.

(a) Work out the angle of the sector.
(b) Work out the area of the sector. Give your answer to (b) in terms of π.

(a) Arc length = $\frac{\theta}{360} \times 2\pi \times 7$, so $\frac{\theta}{360} \times 14 = \frac{7}{2}$

$\theta = \frac{7}{2} \times \frac{360}{14} = 90°$

(b) Area = $\frac{90}{360} \times \pi \times 7^2 = \frac{49}{4}\pi$ cm^2

Answer: (a) 90° (b) $\frac{49}{4}\pi$ cm^2

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6 Sample space and conditional probability

A spinner numbered 1 to 4 and a spinner numbered 1 to 5 are each thrown once. The first score and the second score are recorded.

(a) Work out the probability that the product is greater than 8.

(b) Given that at least one number is 3, work out the probability that the product is greater than 8.

(a) There are 4 × 5 = 20 equally likely outcomes, and 7 of them satisfy: the product is greater than 8

Probability = $\frac{7}{20}$

(b) 8 outcomes satisfy: at least one number is 3. Of these, 4 also satisfy: the product is greater than 8

Probability = $\frac{1}{2}$

Answer: (a) $\frac{7}{20}$ (b) $\frac{1}{2}$

7 Bearings with the sine and cosine rules

A boat leaves a harbour P and travels 69 km on a bearing of 165° to Q. (Calculator)
It then travels 74 km on a bearing of 300° to R.

(a) Work out the distance PR.

(b) Work out the bearing of R from P.

Give distances to 1 decimal place and the bearing to the nearest degree.

(a) The bearing of P from Q is 165° + 180° = 345°

Angle PQR = 45° (the difference between the bearings 345° and 300°)

$PR^2 = 69^2 + 74^2 - 2 \times 69 \times 74 \times \cos 45° = 3016.0$, so PR = 54.9 km

(b) $\cos(QPR) = \frac{69^2 + PR^2 - 74^2}{2 \times 69 \times PR}$, so angle QPR = 72.3°

R is clockwise of the direction PQ, so the bearing is 165° + 72° = 237°

Answer: (a) 54.9 km (b) 237°

8 Proof with indices

Show that $2^{m+3} - 2^m$ is a multiple of 7 for all integer values of m.

$2^{m+3} = 2^m \times 2^3$

$2^{m+3} - 2^m = 2^m(2^3 - 1)$

$2^3 - 1 = 7$

$= 7 \times 2^m$, which is a multiple of 7

Answer: 7×2^m

9 Functions that are their own inverse

$f(x) = \frac{6x + 5}{x - 6}$ where $x \neq 6$

(a) Show that ff(x) = x.

(b) Write down $f^{-1}(x)$.

(a) Replace x in f(x) by f(x), then multiply the top and the bottom by (x − 6)

$ff(x) = \frac{6(6x + 5) + 5(x - 6)}{(6x + 5) - 6(x - 6)}$

Top = 36x + 30 + 5x − 30 = 41x

Bottom = 6x + 5 − 6x + 36 = 41

$ff(x) = \frac{41x}{41} = x$

(b) ff(x) = x, so f is its own inverse: $f^{-1}(x) = \frac{6x + 5}{x - 6}$

Answer: (b) $f^{-1}(x) = \frac{6x + 5}{x - 6}$

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10 Percentages with an unknown rate

A population is increased by x% in one year and then increased by a further x% the next year.
Over the two years the population increases by 10.25%.
Work out the value of x.

Yearly multiplier = $1 + \frac{x}{100}$

Two years: $(1 + \frac{x}{100})^2 = 1.1025$

Take the square root: $1 + \frac{x}{100} = 1.05$

$\frac{x}{100} = 0.05$, so $x = 5$

Answer: x = 5