Day 90 **S Tier 10-a-day: Model Solutions**

1 At least one

The probability that a bus is more than 5 minutes late on any morning is $\frac{3}{20}$.

Events on different occasions are independent.
Work out the smallest number of mornings needed so that the probability of this happening at least once is greater than 0.5. (Calculator)

P(at least once in n mornings) = $1 - \frac{17}{20}^{n}$

n = 4: $1 - \frac{17}{20}^{4} = 0.4780$

n = 5: $1 - \frac{17}{20}^{5} = 0.5563$

0.5563 is the first value above 0.5

Answer: 5

2 Error intervals

The length of a rope is 2850 m, correct to the nearest 50.
Write down the error interval for the length of a rope, using l.

The value has been rounded to the nearest 50
Half of this is 25, so go 25 below and 25 above
2850 − 25 = 2825 and 2850 + 25 = 2875
$2825 \leq l < 2875$

Answer: $2825 \leq l < 2875$

3 Inequalities

(a) Solve $-2x > -2$
(b) n is an integer. Write down all the values of n that satisfy
$-9 < 4n - 1 \leq 7$

(a) Subtract 0: $-2x > -2$
Divide by -2 and reverse the inequality sign: $x < 1$
(b) Add 1: $-8 < 4n \leq 8$
Divide by 4: $-2 < n \leq 2$
Integers: $-1, 0, 1, 2$

Answer: (a) $x < 1$ (b) $-1, 0, 1, 2$

4 Circle theorems: angle at the centre

A, B and C are points on a circle, centre O. Angle ACB = 28°.
(a) Work out the size of angle AOB.
(b) Work out the size of angle OAB.
The diagram is not drawn accurately.

(a) The angle at the centre is twice the angle at the circumference: 2 × 28 = 56°
(b) OA = OB (radii), so triangle OAB is isosceles
Angle OAB = (180 − 56) ÷ 2 = 62°

Answer: (a) 56° (b) 62°

5 Transformations of graphs

The point P(−4, 2) lies on the graph of y = f(x).
Write down the coordinates of the image of P on the graph of
(a) y = 2f(x) (b) y = f(x − 2)

(a) y = 2f(x) is a vertical stretch with scale factor 2, so the y coordinate is doubled: (−4, 4)
(b) y = f(x − 2) is a translation of 2 to the right: (−2, 2)

Answer: (a) (−4, 4) (b) (−2, 2)

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6 Similar triangles with algebra

In the diagram, DE is parallel to BC.
AD = x cm, DB = (x + 4) cm, DE = 4 cm and BC = 10 cm.
(a) Find the value of x.
(b) Work out the length of AB.
The diagram is not drawn accurately.

(a) Triangles ADE and ABC are similar, so $\frac{AD}{AB} = \frac{DE}{BC}$

$\frac{x}{2x + 4} = \frac{4}{10}$
$10x = 4(2x + 4)$
$10x = 8x + 16$
$2x = 16$
$x = 8$
(b) $AB = 2x + 4 = 20$ cm
Answer: (a) x = 8 (b) 20 cm

7 Squares, cubes and prime factors

Find the smallest positive integer k such that 12 × k is a cube number.

$12 = 2^2 \times 3$
For a cube number every power must be a multiple of 3
Multiply by 2×3^2 to get $2^3 \times 3^3$
k = 18
Answer: k = 18

8 Venn diagrams with algebra

The Venn diagram shows how many of 39 students study French (F) or Spanish (S).
(a) Work out the value of x.
(b) A student is chosen at random from those who study Spanish. Work out the probability that this student also studies French.

(a) Total: (x + 2) + x + (2x) + 5 = 39
4x + 7 = 39, so x = 8
(b) Students in the second circle: x + 2x = 8 + 16 = 24, of whom 8 are in both
Probability = $\frac{1}{3}$
Answer: (a) x = 8 (b) $\frac{1}{3}$

9 Proof with indices

Show that $2^{m+2} + 2^m$ is always a multiple of 5.

$2^{m+2} = 2^m \times 2^2$
$2^{m+2} + 2^m = 2^m(2^2 + 1)$
$2^2 + 1 = 5$
$= 5 \times 2^m$, which is a multiple of 5
Answer: 5×2^m

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10 Exact trigonometric ratios from a given ratio

x is an obtuse angle and $\sin x = \frac{7}{9}$.

(a) Find the exact value of cos x.

(b) Find the exact value of tan x. Give each answer as a fraction or a surd. Do not use a calculator.

(a) $\cos^2 x = 1 - (\frac{7}{9})^2 = \frac{32}{81}$

x is obtuse, so cos x is negative: $\cos x = -\frac{4\sqrt{2}}{9}$

(b) $\tan x = \frac{\sin x}{\cos x} = \frac{7}{9} \div (-\frac{4\sqrt{2}}{9}) = -\frac{7\sqrt{2}}{8}$

Answer: (a) $-\frac{4\sqrt{2}}{9}$ (b) $-\frac{7\sqrt{2}}{8}$