Day 9 **S Tier 10-a-day: Model Solutions**

1 Angles in polygons

The interior angles of a hexagon are
$x°$, $(x + 15)°$, $(x + 20)°$, $(x + 30)°$, $(2x - 15)°$, $(2x - 10)°$
(a) Work out the value of x.
(b) Work out the size of the largest angle.

(a) Sum of interior angles = $(6 - 2) \times 180 = 720°$
$8x + 40 = 720$, so $8x = 680$
$x = 85$
(b) The angles are 85°, 100°, 105°, 115°, 155°, 160°
The largest is 160°
Answer: (a) x = 85 (b) 160°

2 Substitution

Work out the value of $(p - q)^2 - (p + q)^2$
when $p = -4$, $q = 2$ and $r = -1$.

Substitute the values: $(-4 - 2)^2 - (-4 + 2)^2$
$= 36 - 4$
$= 32$
Answer: 32

3 Tests of divisibility

The 4-digit number 831k is a multiple of 4.
k is a single digit.
Find all the possible values of k.

A number is a multiple of 4 if its last two digits form a multiple of 4
Last two digits: 1k
Multiples of 4 of this form: 12, 16
Answer: k = 2 or 6

4 Laws of indices with algebra

Simplify $(8x^6y^9)^{\frac{2}{3}}$

Raise each part to the power $\frac{2}{3}$

$8^{\frac{2}{3}} = (\sqrt[3]{8})^2 = 2^2 = 4$

$(x^6)^{\frac{2}{3}} = x^4$ and $(y^9)^{\frac{2}{3}} = y^6$
$= 4x^4y^6$
Answer: $4x^4y^6$

5 Recurring decimals

Write $0.8\dot{6}\dot{9}$ as a fraction in its simplest form.

Let $x = 0.8696...$
$10x = 8.6969...$
$1000x = 869.6969...$
Subtract: $990x = 861$
$x = \frac{861}{990} = \frac{287}{330}$
Answer: $\frac{287}{330}$

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6 Cumulative frequency graphs

The cumulative frequency graph gives information about the masses of 80 apples picked from a tree.
(a) Find an estimate for the median and the interquartile range.
(b) Estimate how many of the 80 had a mass greater than 55.

(a) Median at cumulative frequency 40, so median = 30
Lower quartile at 20, so LQ = 20. Upper quartile at 60, so UQ = 40
Interquartile range = 40 − 20 = 20
(b) At 55, cumulative frequency = 71
Number greater = 80 − 71 = 9
Answer: (a) median 30, IQR 20 (b) 9

7 Composite functions: fg and gf

$f(x) = 4x + 4$ and $g(x) = x^2 - 3$
Solve $fg(x) = gf(x)$

$fg(x) = f(x^2 - 3) = 4x^2 - 8$
$gf(x) = g(4x + 4) = (4x + 4)^2 - 3 = 16x^2 + 32x + 13$
Set them equal and rearrange: $12x^2 + 32x + 21 = 0$
Factorise: $(2x + 3)(6x + 7) = 0$
$x = -\frac{3}{2}$ or $x = -\frac{7}{6}$

Answer: $x = -\frac{3}{2}$ or $x = -\frac{7}{6}$

8 Probability and quadratic equations

A bag contains some counters. 4 of them are green and the rest are white.
Two counters are taken at random without replacement.
The probability that both are green is $\frac{1}{6}$.
Work out how many counters are white.

Let there be n counters altogether
$P(\text{both green}) = \frac{4}{n} \times \frac{3}{n-1} = \frac{12}{n(n-1)}$
$\frac{12}{n(n-1)} = \frac{1}{6}$, so $n(n-1) = 72$, giving $n^2 - n - 72 = 0$
$(n - 9)(n + 8) = 0$, so $n = 9$ (n cannot be negative)
Number of counters that are white = 9 − 4 = 5
Answer: 5

9 Loci and regions

ABCD is a rectangle with AB = 4 cm and AD = 5 cm.
A point P inside the rectangle is such that AP ≤ 4 cm and P is nearer to A than to B.
Work out the area of the region in which P can lie. Give your answer in terms of π and surds.

Nearer to A than to B: P is on the A side of the perpendicular bisector of AB, a distance 2 cm from A
AP ≤ 4 cm: P is inside the quarter circle of radius 4 cm centred at A
The bisector meets the arc where $\cos\theta = \frac{2}{4}$, so $\theta = 60°$ at A. The region is a right-angled triangle plus a sector of angle 30°
Triangle: base 2 cm, height $4 \sin 60° = 2\sqrt{3}$ cm, area $= \frac{1}{2} \times 2 \times 2\sqrt{3} = 2\sqrt{3}$
Sector of angle 30° : $\frac{30}{360} \times \pi \times 4^2 = \frac{4}{3}\pi$
Area $= 2\sqrt{3} + \frac{4}{3}\pi$ cm^2
Answer: $(2\sqrt{3} + \frac{4}{3}\pi)$ cm^2

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10 Bounds of roots

The volume of a cube is 137 cm^3, correct to the nearest cm^3.
Work out the upper bound of the edge length of the cube.
Give your answer correct to 3 significant figures. (Calculator)

$136.5 \leq$ volume < 137.5

Upper bound of edge = $\sqrt[3]{137.5}$
= 5.16140... = 5.16 (3 s.f.)
Answer: 5.16 cm