Day 88 **S Tier 10-a-day: Model Solutions**

1 Angles in polygons

Each interior angle of a regular polygon is 14 times the size of each exterior angle.
Work out the number of sides of the polygon.

Let each exterior angle be e. Then each interior angle is 14e.
Interior angle + exterior angle = 180°, so 14e + e = 180°
e = 180° ÷ 15 = 12°
Number of sides = 360° ÷ 12° = 30
Answer: 30

2 Estimated mean with an unknown frequency

The table gives information about the times taken by some people to travel to work.
An estimate for the mean is 30.2.
Work out the value of x.

Mid-points: 15, 25, 35, 45
Estimated mean = $\Sigma fm \div \Sigma f$, so $(880 + 35x) \div (32 + x) = 30.2$
$880 + 35x = 30.2(32 + x) = 966.4 + 30.2x$
$(35 - 30.2)x = 966.4 - 880$
$4.8x = 86.4$, so $x = 18$
Answer: x = 18

3 Changing the subject

Make l the subject of $T = \frac{7}{\sqrt{l}}$

Multiply by $\sqrt{l}$: $T\sqrt{l} = 7$

Divide by T: $\sqrt{l} = \frac{7}{T}$

Square both sides: $l = \frac{49}{T^2}$

Answer: $l = \frac{49}{T^2}$

4 Perpendicular bisector

A is the point (−1, 2) and B is the point (3, −4).
Find an equation of the perpendicular bisector of AB. Give your answer in the form ax + by = c, where a, b and c are integers.

Midpoint of AB = (1, −1)

Gradient of AB = $-\frac{3}{2}$

Perpendicular gradient = $\frac{2}{3}$

$y - (-1) = \frac{2}{3}(x - 1)$

Rearrange to integer coefficients: $2x - 3y = 5$
Answer: $2x - 3y = 5$

5 Standard form

Work out $(6.8 \times 10^7) + (4.5 \times 10^4)$
Give your answer in standard form.

Write the first number as a multiple of 10^4: $6.8 \times 10^7 = 6800 \times 10^4$
$(6800 + 4.5) \times 10^4 = 6804.5 \times 10^4$
$= 6.8045 \times 10^7$
Answer: 6.8045×10^7

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6 Radius from chord and height of arc

A circular arc has a chord AB of length 48 cm.
The greatest distance between the chord and the arc is 12 cm.
Work out the radius of the circle.

Let the radius be r. The centre O is on the line through the midpoint M of AB and the highest point of the arc, so OM = r − 12

AM = 48 ÷ 2 = 24. Pythagoras in triangle OMA: $r^2 = (r - 12)^2 + 24^2$

$r^2 = r^2 - 24r + 144 + 576$

24r = 720, so r = 30 cm

Answer: 30 cm

7 Powers of standard form

Work out $(2.3 \times 10^{-3})^2$
Give your answer in standard form.

$(2.3 \times 10^{-3})^2 = 2.3^2 \times (10^{-3})^2$

$2.3^2 = 5.29$ and $(10^{-3})^2 = 10^{-6}$

$= 5.29 \times 10^{-6}$

Answer: 5.29×10^{-6}

8 Exact lengths and areas

A rectangle has area $(25 + 11\sqrt{5})$ cm^2 and length $(2 + \sqrt{5})$ cm.
Find the width of the rectangle.
Give your answer in the form $a + b\sqrt{5}$

Width = area ÷ length = $\dfrac{25 + 11\sqrt{5}}{2 + \sqrt{5}}$

Multiply top and bottom by $(2 - \sqrt{5})$:

Numerator: $(25 + 11\sqrt{5})(2 - \sqrt{5}) = -5 - 3\sqrt{5}$

Denominator: 4 − 5 = −1

Width = $5 + 3\sqrt{5}$

Answer: $5 + 3\sqrt{5}$ cm

9 Algebraic proof (harder)

Prove algebraically that the difference between the cubes of any two consecutive integers is always an odd number.

Let the integers be m and m + 1.

$(m + 1)^3 - m^3 = m^3 + 3m^2 + 3m + 1 - m^3 = 3m^2 + 3m + 1$

$= 3m(m + 1) + 1$

m(m + 1) is even, so 3m(m + 1) is even, and adding 1 gives an odd number

Answer: Proved

10 Histograms with an unmarked scale

The histogram shows the masses of parcels delivered by a courier.
30 of them had a mass between 40 and 70 kg.
(a) How many had a mass between 0 and 10 kg?
(b) How many parcels are there in total?

Frequency is proportional to the area of a bar

40 to 70: bar is 1 squares high and 30 wide, area 30, which represents 30

So 1 unit of area represents 30 ÷ 30 = 1

(a) 0 to 10: 6 × 10 = 60 units of area, so 60 × 1 = 60

(b) Total area = 60 + 50 + 20 + 30 = 160, so total = 160 × 1 = 160

Answer: (a) 60 (b) 160