Day 87 **S Tier 10-a-day: Model Solutions**

1 Error intervals

The length of a rope is 3.2 m, correct to 1 decimal place.
Write down the error interval for the length of a rope, using l.

The value has been rounded to the nearest 0.1
Half of this is 0.05, so go 0.05 below and 0.05 above
$3.2 - 0.05 = 3.15$ and $3.2 + 0.05 = 3.25$
$3.15 \leq l < 3.25$
Answer: $3.15 \leq l < 3.25$

2 Weighted percentages

The ratio of juniors to seniors in a school is 5 : 3.
70% of the juniors and 50% of the seniors play a sport.
What percentage of all of these students play a sport?

Imagine 160 students in total: 100 juniors and 60 seniors
70% of 100 = 70 and 50% of 60 = 30
Together: 100 out of 160
$100 \div 160 \times 100 = 62.5\%$
Answer: 62.5%

3 Inequalities

(a) Solve $6 - 5x > 11$
(b) n is an integer. Write down all the values of n that satisfy
$-1 \leq 3n - 1 < 11$

(a) Subtract 6: $-5x > 5$
Divide by -5 and reverse the inequality sign: $x < -1$
(b) Add 1: $0 \leq 3n < 12$
Divide by 3: $0 \leq n < 4$
Integers: 0, 1, 2, 3
Answer: (a) $x < -1$ (b) 0, 1, 2, 3

4 Exact trigonometric values

Work out the exact value of $\tan 30° - \cos 60°$
Give your answer in its simplest form, as a fraction or surd. Do not use a calculator.

$\tan 30° - \cos 60° = \frac{\sqrt{3}}{3} - \frac{1}{2}$

$= -\frac{1}{2} + \frac{\sqrt{3}}{3}$

Answer: $-\frac{1}{2} + \frac{\sqrt{3}}{3}$

5 Line of best fit

The scatter graph shows distance from town (x) and price (y).
The line of best fit passes through (4, 34) and (8, 28).
(a) Find its equation in the form $y = mx + c$.
(b) Explain what its gradient tells you.

(a) Gradient $m = \frac{28 - 34}{8 - 4} = \frac{-6}{4} = -1.5$
$y = -1.5x + c$: using (4, 34), $34 = -1.5 \times 4 + c$, so $c = 40$
$y = -1.5x + 40$
(b) For each extra kilometre from the town, the house price (in £10 000s) goes down by 1.5
Answer: (a) $y = -1.5x + 40$ (b) For each extra kilometre from the town, the house price (in £10 000s) goes down by 1.5

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6 Transformations of graphs

The point P(2, −5) lies on the graph of y = f(x).
Write down the coordinates of the image of P on the graph of
(a) y = f(x − 5) (b) y = f(x) + 5

(a) y = f(x − 5) is a translation of 5 to the right: (7, −5)
(b) y = f(x) + 5 is a translation of 5 up: (2, 0)
Answer: (a) (7, −5) (b) (2, 0)

7 Volume and surface area of composite solids

A solid is made from a hemisphere joined to a cone. Both have radius 8 cm and the slant height of the cone is 10 cm.
Work out the total surface area of the solid. Give your answer in terms of π.

Curved surface area of cone = $\pi \times r \times l = \pi \times 8 \times 10 = 80\pi$
Surface area of hemisphere = $2\pi r^2 = 2 \times \pi \times 8^2 = 128\pi$
Total = $80\pi + 128\pi = 208\pi$ cm^2
(The flat circle where the two solids join is not part of the surface.)
Answer: 208π cm^2

8 Exact lengths and areas

A rectangle has area $(9 + 4\sqrt{5})$ cm^2 and length $(2 + \sqrt{5})$ cm.
Find the width of the rectangle.
Give your answer in the form $a + b\sqrt{5}$

Width = area ÷ length = $\dfrac{9 + 4\sqrt{5}}{2 + \sqrt{5}}$

Multiply top and bottom by $(2 - \sqrt{5})$:
Numerator: $(9 + 4\sqrt{5})(2 - \sqrt{5}) = -2 - \sqrt{5}$
Denominator: $4 - 5 = -1$
Width = $2 + \sqrt{5}$
Answer: $2 + \sqrt{5}$ cm

9 Algebraic proof (harder)

Show algebraically that k(k + 1)(k + 2)(k + 3) + 1 is a square number for all positive integer values of k.

$k(k + 3) = k^2 + 3k$ and $(k + 1)(k + 2) = k^2 + 3k + 2$
Let $m = k^2 + 3k$. Then the expression is $m(m + 2) + 1$
$= m^2 + 2m + 1 = (m + 1)^2$
$= (k^2 + 3k + 1)^2$, which is the square of an integer
Answer: Proved

10 Vectors in a triangle

OAB is a triangle. OA = **a** and OB = **b**.
M is on OA with OM : MA = 2 : 1. N is on AB with AN : NB = 1 : 4.
(a) Find ON in terms of **a** and **b**.
(b) Find MN in terms of **a** and **b**.
The diagram is not drawn accurately.

(a) AB = $-\mathbf{a} + \mathbf{b}$, so AN = $\frac{1}{5}(\mathbf{b} - \mathbf{a})$

ON = OA + AN = $\mathbf{a} + \frac{1}{5}(\mathbf{b} - \mathbf{a}) = \frac{4}{5}\mathbf{a} + \frac{1}{5}\mathbf{b}$

(b) OM = $\frac{2}{3}\mathbf{a}$

MN = ON − OM = $\frac{4}{5}\mathbf{a} + \frac{1}{5}\mathbf{b} - \frac{2}{3}\mathbf{a} = \frac{2}{15}\mathbf{a} + \frac{1}{5}\mathbf{b}$

Answer: (a) $\frac{4}{5}\mathbf{a} + \frac{1}{5}\mathbf{b}$ (b) $\frac{2}{15}\mathbf{a} + \frac{1}{5}\mathbf{b}$