Day 86 **S Tier 10-a-day: Model Solutions**

1 Angle in a semicircle

AB is a diameter of a circle, centre O. C is a point on the circle. Angle BAC = 33°.
(a) Work out the size of angle ACB.
(b) Work out the size of angle COB.
The diagram is not drawn accurately.

(a) The angle in a semicircle is 90°, so angle ACB = 90°
(b) OA = OC (radii), so triangle OAC is isosceles and angle OCA = 33°
Angle COB is an exterior angle of triangle OAC: 33 + 33 = 66°
(or: the angle at the centre is twice the angle at the circumference)
Answer: (a) 90° (b) 66°

2 Rational and irrational numbers

Here are six numbers.

$(\sqrt{6}+2)(\sqrt{6}-2)$, $\sqrt{49}$, $\sqrt{8}\times\sqrt{2}$, $(1+\sqrt{2})^2$, $\sqrt{3}\times\sqrt{5}$, $\frac{\sqrt{20}}{\sqrt{5}}$

Write down the rational numbers.

Simplify each number:
$(\sqrt{6}+2)(\sqrt{6}-2) = 6-4 = 2$; $\sqrt{49} = 7$; $\sqrt{8}\times\sqrt{2} = \sqrt{16} = 4$
$(1+\sqrt{2})^2 = 3+2\sqrt{2}$; $\sqrt{3}\times\sqrt{5} = \sqrt{15}$; $\frac{\sqrt{20}}{\sqrt{5}} = \sqrt{4} = 2$
Answer: $(\sqrt{6}+2)(\sqrt{6}-2)$, $\sqrt{49}$, $\sqrt{8}\times\sqrt{2}$, $\frac{\sqrt{20}}{\sqrt{5}}$

3 Inequalities

(a) Solve $-2-2x > 6$
(b) n is an integer. Write down all the values of n that satisfy
$-14 < 3n+1 \leq 7$

(a) Add 2: $-2x > 8$
Divide by -2 and reverse the inequality sign: $x < -4$
(b) Subtract 1: $-15 < 3n \leq 6$
Divide by 3: $-5 < n \leq 2$
Integers: $-4, -3, -2, -1, 0, 1, 2$
Answer: (a) $x < -4$ (b) $-4, -3, -2, -1, 0, 1, 2$

4 Compound interest

Amir invests £2,000.
The account pays compound interest of 4% a year for the first 2 years,
then 3% a year for the next 3 years.
Work out the total amount after 5 years, to the nearest penny. (Calculator)

Multipliers: 1.04 for 2 years, then 1.03 for 3 years
$£2{,}000 \times 1.04^2 \times 1.03^3$
= £2,363.79
Answer: £2,363.79

5 Sample space and conditional probability

A fair 4-sided dice and a fair 6-sided dice are each thrown once. The first score and the second score are recorded.
(a) Work out the probability that the two numbers are different.
(b) Given that at least one number is 3, work out the probability that the two numbers are different.

(a) There are 4 × 6 = 24 equally likely outcomes, and 20 of them satisfy: the two numbers are different
Probability = $\frac{5}{6}$
(b) 9 outcomes satisfy: at least one number is 3. Of these, 8 also satisfy: the two numbers are different
Probability = $\frac{8}{9}$
Answer: (a) $\frac{5}{6}$ (b) $\frac{8}{9}$

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6 Solving trigonometric equations

Solve 4 tan x + 5 = 0 for 0° ≤ x ≤ 360°. (Calculator)
Give your answers correct to 1 decimal place.

$\tan x = -\frac{5}{4}$

Related acute angle: $\tan^{-1}(\frac{5}{4}) = 51.3°$

tangent is negative in the 2nd and 4th quadrants, so x = 180° − 51.3° and x = 360° − 51.3°
x = 128.7° and x = 308.7°
Answer: x = 128.7° or x = 308.7°

7 Exponential graphs

The diagram shows the graph of $y = ka^x$, where k and a are constants and a > 0.
The curve passes through the points (1, 24) and (3, 216).
Find the value of k and the value of a.

Substitute the points: $k\,a^1 = 24$ and $k\,a^3 = 216$

Divide: $a^2 = \frac{216}{24} = 9$

a = 3 (positive, since a > 0)
From the first point: k × 3 = 24, so k = 8
Answer: k = 8, a = 3

8 Ratio and money transferred

Beth and Kira share some money in the ratio 2 : 7.
Beth gives Kira £3.
The ratio of their money is now 1 : 5.
How much money do they have altogether?

Let the amounts be 2k and 7k, so the total is 9k

Beth now has 2k − 3, which is $\frac{1}{6}$ of the total

6(2k − 3) = 1 × 9k
12k − 18 = 9k, so 3k = 18 and k = 6
Total = 9 × 6 = £54
Answer: £54

9 Trigonometric graphs

The diagram shows the graph of y = a sin x° + c for 0 ≤ x ≤ 360.
Some points on the graph are labelled.
Find the value of a and the value of c.

At x = 90, sin x° = 1, so a + c = −6
At x = 270, sin x° = −1, so −a + c = −8
Add: 2c = −14, so c = −7
Subtract: 2a = 2, so a = 1
Answer: a = 1, c = −7

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10 Probability and quadratic equations

A bag contains some sweets. 9 of them are red and the rest are yellow.
Two sweets are taken at random without replacement.

The probability that both are red is $\frac{36}{91}$.

Work out how many sweets are yellow.

Let there be n sweets altogether

P(both red) = $\frac{9}{n} \times \frac{8}{n-1} = \frac{72}{n(n-1)}$

$\frac{72}{n(n-1)} = \frac{36}{91}$, so $n(n-1) = 182$, giving $n^2 - n - 182 = 0$

$(n-14)(n+13) = 0$, so $n = 14$ (n cannot be negative)

Number of sweets that are yellow = $14 - 9 = 5$

Answer: 5