Day 85 **S Tier 10-a-day: Model Solutions**

1 Angles with algebra

ABC is an isosceles triangle with AB = AC.
Angle BAC = (3x + 20)°
Angle ABC = (2x + 38)°
(a) Work out the value of x.
(b) Work out the size of angle BAC.

(a) Angles ABC and ACB are equal, and the angles in a triangle add up to 180°
(3x + 20) + 2(2x + 38) = 180
7x + 96 = 180, so 7x = 84
x = 12
(b) Angle BAC = 3 × 12 + 20 = 56°
Answer: (a) x = 12 (b) 56°

2 Inequalities

(a) Solve $3 - 2x \leq -17$
(b) n is an integer. Write down all the values of n that satisfy
$-11 \leq 2n - 5 < -5$

(a) Subtract 3: $-2x \leq -20$
Divide by -2 and reverse the inequality sign: $x \geq 10$
(b) Add 5: $-6 \leq 2n < 0$
Divide by 2: $-3 \leq n < 0$
Integers: $-3, -2, -1$
Answer: (a) $x \geq 10$ (b) $-3, -2, -1$

3 Cyclic quadrilaterals

A, B, C and D are points on a circle. Angle DAB = (2x + 20)°, angle BCD = (3x + 45)° and angle ABC = (2x + 30)°.
(a) Work out the value of x.
(b) Work out the size of angle ADC.
The diagram is not drawn accurately.

(a) Opposite angles of a cyclic quadrilateral add up to 180°
(2x + 20) + (3x + 45) = 180
5x + 65 = 180, so 5x = 115
x = 23
(b) Angle ABC = 2 × 23 + 30 = 76°
Angle ADC = 180 − 76 = 104°
Answer: (a) x = 23 (b) 104°

4 Fraction problems

A bag contains sweets.
$\frac{3}{5}$ of the sweets are orange and $\frac{1}{4}$ of the sweets are lemon.
The rest of the sweets, 18 of them, are strawberry.
How many sweets are in the bag?

Fraction that are strawberry = $1 - \frac{3}{5} - \frac{1}{4} = \frac{3}{20}$

$\frac{3}{20}$ of the total is 18

Total = $18 \div \frac{3}{20} = 120$

Answer: 120

Day 85 **S Tier 10-a-day: Model Solutions**

5 Inverse proportion

y is inversely proportional to the square root of x.
When x = 9, y = 20.
Find the value of y when x = 25.

$y = \frac{k}{\sqrt{x}}$

$20 = \frac{k}{3}$, so $k = 20 \times 3 = 60$

$y = \frac{60}{5} = 12$

Answer: y = 12

6 Algebraic proof

Prove that the sum of the squares of any five consecutive integers is always a multiple of 5.

Let the integers be $n - 2$, $n - 1$, n, $n + 1$ and $n + 2$.

$(n - 2)^2 + (n - 1)^2 + n^2 + (n + 1)^2 + (n + 2)^2 = 5n^2 + 2 \times 1 + 2 \times 4$

$= 5n^2 + 10 = 5(n^2 + 2)$

This is 5 × an integer, so it is a multiple of 5

Answer: Proved

7 Index equations

Solve $16^{x+4} = 4^4$

Write both sides as powers of 2: $16 = 2^4$ and $4 = 2^2$

Equate the powers: $4x + 16 = 8$

$4x = -8$, so $x = -2$

Answer: x = −2

8 Joint and inverse variation

F is directly proportional to m and inversely proportional to the square of d.
When m = 6 and d = 6, F = 2.
Work out the value of F when m = 4 and d = 1.

$F = \frac{km}{d^2}$

$2 = \frac{k \times 6}{36}$, so $k = 2 \times 36 \div 6 = 12$

$F = \frac{12 \times 4}{1} = 48$

Answer: F = 48

Day 85 **S Tier 10-a-day: Model Solutions**

9 Vectors: intersecting lines

OAB is a triangle. OA = $\mathbf{a}$ and OB = $\mathbf{b}$.
M is on OA with OM : MA = 3 : 1. N is on OB with ON : NB = 1 : 4. AN and BM intersect at X.
(a) Find BM in terms of $\mathbf{a}$ and $\mathbf{b}$.
(b) Find OX in terms of $\mathbf{a}$ and $\mathbf{b}$.
The diagram is not drawn accurately.

(a) OM = $\frac{3}{4}\mathbf{a}$, so BM = $-\mathbf{b} + \frac{3}{4}\mathbf{a} = \frac{3}{4}\mathbf{a} - \mathbf{b}$

(b) X is on BM: BX = s BM, so OX = OB + s BM = $\frac{3}{4}s\mathbf{a} + (1 - s)\mathbf{b}$

X is on AN: OX = OA + t AN = OA + t(ON − OA) = $(1 - t)\mathbf{a} + \frac{1}{5}t\mathbf{b}$

Equate: $\frac{3}{4}s = 1 - t$ and $1 - s = \frac{1}{5}t$

Solving: $s = \frac{16}{17}$ and $t = \frac{5}{17}$

OX = $\frac{12}{17}\mathbf{a} + \frac{1}{17}\mathbf{b}$

Answer: (a) $\frac{3}{4}\mathbf{a} - \mathbf{b}$ (b) OX = $\frac{12}{17}\mathbf{a} + \frac{1}{17}\mathbf{b}$

10 Estimating the median from grouped data

The table gives information about 90 masses.
Work out an estimate for the median.
Assume the values in each class are evenly spread.

Cumulative frequencies: 27, 57, 65, 71, 86, 90
The median is the value at position 90 ÷ 2 = 45
It is in the class 20 to 30, with 27 values before it, so we need 18 more of the 30 values in this class
Median ≈ $20 + \frac{18}{30} \times 10 = 26$

Answer: 26