Day 84 **S Tier 10-a-day: Model Solutions**

1 Bearings with the sine and cosine rules

A boat leaves a harbour P and travels 83 km on a bearing of 135° to Q. (Calculator)
It then travels 97 km on a bearing of 285° to R.
(a) Work out the distance PR.
(b) Work out the bearing of R from P.
Give distances to 1 decimal place and the bearing to the nearest degree.

(a) The bearing of P from Q is 135° + 180° = 315°
Angle PQR = 30° (the difference between the bearings 315° and 285°)
$PR^2 = 83^2 + 97^2 - 2 \times 83 \times 97 \times \cos 30° = 2353.3$, so PR = 48.5 km

(b) $\cos(QPR) = \frac{83^2 + PR^2 - 97^2}{2 \times 83 \times PR}$, so angle QPR = 91.2°

R is clockwise of the direction PQ, so the bearing is 135° + 91° = 226°

Answer: (a) 48.5 km (b) 226°

2 Expanding three brackets

Expand and simplify $(x - 6)^2(x + 3)$

Expand the first two brackets: $(x - 6)^2 = x^2 - 12x + 36$
Multiply by (x + 3): $x(x^2 - 12x + 36) + 3(x^2 - 12x + 36)$
$= x^3 - 9x^2 + 108$

Answer: $x^3 - 9x^2 + 108$

3 Percentage change

A 20% decrease followed by a 15% decrease is equivalent to a single percentage change.
Work out the single percentage change that has the same effect.

Multipliers: 0.8 and 0.85
Overall multiplier = 0.8 × 0.85 = 0.68
0.68 − 1 = −0.32, so an overall decrease of 32%

Answer: 32% decrease

4 Tree diagrams without replacement

A bag contains 5 red sweets, 4 blue sweets and 1 green sweet.
Two sweets are taken at random without replacement.
Work out the probability that (a) they are different colours, (b) both are the same colour.

(a) $P(\text{different}) = 1 - P(\text{same}) = 1 - \frac{16}{45} = \frac{29}{45}$

(b) Add the ways: $\frac{5}{10} \times \frac{4}{9} + \frac{4}{10} \times \frac{3}{9} = \frac{16}{45}$

Answer: (a) $\frac{29}{45}$ (b) $\frac{16}{45}$

5 Transformations of graphs

The graph of y = f(x) has a minimum point at (4, 1).
Write down the coordinates of the turning point of the graph of
(a) y = f(x − 2)
(b) y = f(x − 4) − 1

(a) Start with the turning point (4, 1)
Apply the transformation to the point: (6, 1)
(b) f(x − 4) changes the x-coordinate to 8, and f(x) − 1 changes the y-coordinate to 0
Turning point: (8, 0)

Answer: (a) (6, 1) (b) (8, 0)

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6 Forming and solving quadratic equations

A rectangle has length (3x + 5) cm and width (x + 4) cm.
The area of the rectangle is 348 cm^2.
(a) Show that $3x^2 + 17x - 328 = 0$
(b) Work out the perimeter of the rectangle.

(a) $(3x + 5)(x + 4) = 348$, so $3x^2 + 17x + 20 = 348$
$3x^2 + 17x - 328 = 0$
(b) $3x^2 + 17x - 328 = 0$: $(x - 8)(3x + 41) = 0$
$x = 8$ ($x = -\frac{41}{3}$ is not possible as lengths are positive)
Length = 29 cm and width = 12 cm, so perimeter = 2 × (29 + 12) = 82 cm
Answer: (b) 82 cm

7 Reverse percentages

The price of a bicycle is increased by 5%.
The new price is then decreased by 8%.
The final price is £966.
Work out the original price of the bicycle.

Multipliers: 1.05, 0.92
Overall multiplier = 1.05 × 0.92 = 0.966
Original = final ÷ multiplier = £966 ÷ 0.966
= £1,000
Answer: £1,000

8 Probability from a frequency tree

In a population of 10,000 people, 1% have a certain disease.
A test gives a positive result for 80% of people who have the disease and for 4% of people who do not.
A person is chosen at random and tests positive.
Work out the probability that they have the disease.

1% of 10,000 = 100 have the disease and 9,900 do not
Flagged and genuine: 100 × 80% = 80
Flagged but not: 9,900 × 4% = 396
Total flagged or positive = 80 + 396 = 476
Probability = 80 ÷ 476 = $\frac{20}{119}$
Answer: $\mathbf{\frac{20}{119}}$

9 Area and quadratic equations

The diagram shows a rectangle of length (3x + 1) cm, width (x − 1) cm and area 39 cm^2.
(a) Show that $3x^2 - 2x - 40 = 0$.
(b) Work out the perimeter of the rectangle.

(a) $(3x + 1)(x - 1) = 39$
Expand: $3x^2 - 2x - 1 = 39$, so $3x^2 - 2x - 40 = 0$
(b) Factorise: $(x - 4)(3x + 10) = 0$
$x = 4$ ($x = -\frac{10}{3}$ is not possible as a length would be negative)
Length = 13 cm and width = 3 cm
Perimeter = 2 × (13 + 3) = 32 cm
Answer: (b) 32 cm

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10 Percentages with an unknown rate

The price of a television is decreased by x%.
The new price is then increased by x%.
The final price is 1% less than the original price.
Work out the value of x.

Let the original price be 1. Multipliers: $(1 + \frac{x}{100})$ and $(1 - \frac{x}{100})$

$(1 + \frac{x}{100})(1 - \frac{x}{100}) = 1 - \frac{x^2}{10000}$

Final price is 1% less, so the multiplier is 0.99: $1 - \frac{x^2}{10000} = 0.99$

$\frac{x^2}{10000} = 0.01$, so $x^2 = 100$ and $x = 10$

Answer: x = 10