Day 83 **S Tier 10-a-day: Model Solutions**

1 Tangent to a circle

A circle has equation $x^2 + y^2 = 65$.
The point P(−7, 4) lies on the circle.
Find an equation of the tangent to the circle at P.
Give your answer in the form ax + by = c, where a, b and c are integers.

The centre is (0, 0). The gradient of the radius to P is $\frac{4-0}{-7-0} = -\frac{4}{7}$

The tangent is perpendicular to the radius, so its gradient is $\frac{7}{4}$. Using P: $y - 4 = \frac{7}{4}(x + 7)$

Multiply by 4: $4(y - 4) = 7(x + 7)$
Expand and rearrange: $-7x + 4y = 65$
Multiply by −1: $7x - 4y = -65$
Answer: $7x - 4y = -65$

2 Angles with algebra

ABC is an isosceles triangle with AB = AC.
Angle BAC = (4x − 20)°
Angle ABC = (2x + 24)°
(a) Work out the value of x.
(b) Work out the size of angle BAC.

(a) Angles ABC and ACB are equal, and the angles in a triangle add up to 180°
(4x − 20) + 2(2x + 24) = 180
8x + 28 = 180, so 8x = 152
x = 19
(b) Angle BAC = 4 × 19 − 20 = 56°
Answer: (a) x = 19 (b) 56°

3 Time zones

A flight leaves London at 18:40 (London time).
It takes 12 hours 45 minutes.
When it is 12:00 in London it is 20:00 in Tokyo.
At what time, in Tokyo time, does the flight land? Say if it is on a different day.

Arrival in London time = 18:40 + 12 h 45 min = 07:25 the next day
Tokyo is 8 hours ahead of London, so add 8 hours
15:25 the next day
Answer: 15:25 the next day

4 Density of an alloy

An alloy is made by mixing 20 cm^3 of iron with 30 cm^3 of aluminium.
The density of iron is 7.9 g/cm^3. The density of aluminium is 2.7 g/cm^3.
Work out the density of the alloy.

Mass of iron = 20 × 7.9 = 158 g
Mass of aluminium = 30 × 2.7 = 81 g
Total mass = 239 g, total volume = 50 cm^3
Density = 239 ÷ 50 = 4.78 g/cm^3
Answer: 4.78 g/cm^3

5 Alternate segment theorem

PAQ is a tangent to the circle at A. B and C are points on the circle. Angle PAB = 78° and angle CAQ = 32°.
(a) Work out the size of angle ABC.
(b) Work out the size of angle ACB.
The diagram is not drawn accurately.

(a) Angle ABC = angle CAQ = 32° (alternate segment theorem)
(b) Angle ACB = angle PAB = 78° (alternate segment theorem)
Answer: (a) 32° (b) 78°

Day 83 **S Tier 10-a-day: Model Solutions**

6 Inverse functions

$f(x) = \frac{4}{x-5} + 2$

(a) Find $f^{-1}(x)$.

(b) Find the value of $f^{-1}(0)$.

(a) Let $y = \frac{4}{x-5} + 2$, so $y - 2 = \frac{4}{x-5}$

Make x the subject: $x - 5 = \frac{4}{y-2}$

$x = \frac{4}{y-2} + 5$

$f^{-1}(x) = \frac{4}{x-2} + 5$

(b) $f^{-1}(0) = \frac{4}{0-2} + 5 = 3$

Answer: (a) $f^{-1}(x) = \frac{4}{x-2} + 5$ (b) 3

7 Compound interest

Jess invests £1,000.
The account pays compound interest of 6% a year for the first 2 years,
then 3.5% a year for the next 2 years.
Work out the total amount after 4 years, to the nearest penny. (Calculator)

Multipliers: 1.06 for 2 years, then 1.035 for 2 years

$£1{,}000 \times 1.06^2 \times 1.035^2$

= £1,203.63

Answer: £1,203.63

8 Growth and decay

A culture has 800 bacteria.
The number of bacteria increases by 6% each hour.
After how many whole hours will there first be more than 2000 bacteria? (Calculator)

Multiplier each hour = 1.06

After 15 hours: $800 \times 1.06^{15} = 1917$

After 16 hours: $800 \times 1.06^{16} = 2032$

So the first time it is more than 2000 is after 16 hours

Answer: 16 hours

9 Probability and quadratic equations

A bag contains some sweets. 3 of them are red and the rest are yellow.
Two sweets are taken at random without replacement.

The probability that both are red is $\frac{3}{55}$.

Work out how many sweets are yellow.

Let there be n sweets altogether

$P(\text{both red}) = \frac{3}{n} \times \frac{2}{n-1} = \frac{6}{n(n-1)}$

$\frac{6}{n(n-1)} = \frac{3}{55}$, so $n(n-1) = 110$, giving $n^2 - n - 110 = 0$

$(n-11)(n+10) = 0$, so $n = 11$ (n cannot be negative)

Number of sweets that are yellow = $11 - 3 = 8$

Answer: 8

Day 83 **S Tier 10-a-day: Model Solutions**

10 Angles of elevation

A point A is on level ground in line with the base of a cliff. (Calculator)
The angle of elevation of the top of the cliff from A is 31°.
B is a point on the same line, 30 m further from the cliff than A.
The angle of elevation of the top of the cliff from B is 23°.
Work out the height of the cliff. Give your answer to 1 decimal place.

Let the height be h and the distance from A to the base be x. $\tan 31° = \frac{h}{x}$, so $x = h \div \tan 31°$

From B: $\tan 23° = \frac{h}{x + 30}$, so $x + 30 = h \div \tan 23°$

Subtract: $30 = h \div \tan 23° - h \div \tan 31° = h(2.3559 - 1.6643)$
$h = 30 \div 0.69157 = 43.4$ m (1 d.p.)
Answer: 43.4 m