Day 82 **S Tier 10-a-day: Model Solutions**

1 Relative speed

Maya and Nadia live 290 km apart.
At 10:00 they set off towards each other on the same road.
Maya drives at 85 km/h and Nadia drives at 60 km/h.
(a) At what time do they meet?
(b) How far from Maya's home do they meet?

Closing speed = 85 + 60 = 145 km/h
(a) Time = 290 ÷ 145 = 2 hours = 120 minutes, so they meet at 12:00
(b) Distance = 85 × 2 = 170 km
Answer: (a) 12:00 (b) 170 km

2 Equations with indices

Solve $5^x = 25^{x+2}$

Write both sides as powers of 5: $(5^1)^x = (5^2)^{x+2}$
$5^x = 5^{2x+4}$
Equate the powers: $x = 2x + 4$
$-x = 4$
$x = -4$
Answer: $x = -4$

3 Cyclic quadrilaterals

A, B, C and D are points on a circle. Angle DAB = $(3x + 30)°$, angle BCD = $(2x + 25)°$ and angle ABC = $(3x - 5)°$.
(a) Work out the value of x.
(b) Work out the size of angle ADC.
The diagram is not drawn accurately.

(a) Opposite angles of a cyclic quadrilateral add up to 180°
$(3x + 30) + (2x + 25) = 180$
$5x + 55 = 180$, so $5x = 125$
$x = 25$
(b) Angle ABC = $3 \times 25 - 5 = 70°$
Angle ADC = $180 - 70 = 110°$
Answer: (a) x = 25 (b) 110°

4 Fraction problems

A bag contains balloons.
$\frac{3}{8}$ of the balloons are pink and $\frac{1}{5}$ of the balloons are silver.
The rest of the balloons, 51 of them, are gold.
How many pink balloons are there?

Fraction that are gold = $1 - \frac{3}{8} - \frac{1}{5} = \frac{17}{40}$

$\frac{17}{40}$ of the total is 51

Total = $51 \div \frac{17}{40} = 120$

Pink balloons = $\frac{3}{8}$ of 120 = 45

Answer: 45

5 Quadratic inequalities

Solve $x^2 \geq 4$

Rearrange: $x^2 - 4 \geq 0$
Solve $x^2 - 4 = 0$: $(x + 2)(x - 2) = 0$, so $x = -2$ or $x = 2$
The graph is a U shape crossing the x-axis at -2 and 2; it is above the axis outside the roots
$x \leq -2$ or $x \geq 2$
Answer: $x \leq -2$ or $x \geq 2$

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6 Upper and lower bounds

a = 0.79 and b = 0.28,
both correct to 2 decimal places.
Work out the upper bound of a ÷ b.
Give your answer correct to 3 significant figures.

Each value is rounded, so it can be up to 0.005 above or below
$0.785 \leq a < 0.795$; $0.275 \leq b < 0.285$
Biggest answer: biggest a ÷ smallest b = 0.795 ÷ 0.275
= 2.89091... = 2.89 to 3 significant figures
Answer: 2.89

7 Exact trigonometric values

Work out the exact value of cos 45° + sin 30°
Give your answer in its simplest form, as a fraction or surd. Do not use a calculator.

$\cos 45° + \sin 30° = \frac{\sqrt{2}}{2} + \frac{1}{2}$

$= \frac{1}{2} + \frac{\sqrt{2}}{2}$

Answer: $\frac{1}{2} + \frac{\sqrt{2}}{2}$

8 Histograms with an unmarked scale

The histogram shows the ages of visitors to a museum.
50 of them had an age between 10 and 20 years.
(a) How many had an age between 20 and 30 years?
(b) How many visitors are there in total?

Frequency is proportional to the area of a bar
10 to 20: bar is 5 squares high and 10 wide, area 50, which represents 50
So 1 unit of area represents 50 ÷ 50 = 1
(a) 20 to 30: 4 × 10 = 40 units of area, so 40 × 1 = 40
(b) Total area = 60 + 50 + 40 + 140 = 290, so total = 290 × 1 = 290
Answer: (a) 40 (b) 290

9 Exact lengths and areas

A rectangle has area $(4 + 2\sqrt{2})$ cm^2 and length $(3 + 2\sqrt{2})$ cm.
Find the width of the rectangle.
Give your answer in the form $a + b\sqrt{2}$

Width = area ÷ length = $\frac{4 + 2\sqrt{2}}{3 + 2\sqrt{2}}$

Multiply top and bottom by $(3 - 2\sqrt{2})$:
Numerator: $(4 + 2\sqrt{2})(3 - 2\sqrt{2}) = 4 - 2\sqrt{2}$
Denominator: $9 - 8 = 1$
Width = $4 - 2\sqrt{2}$
Answer: $4 - 2\sqrt{2}$ cm

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10 Vectors: intersecting lines

OAB is a triangle. OA = $\mathbf{a}$ and OB = $\mathbf{b}$.
M is on OA with OM : MA = 3 : 1. N is on OB with ON : NB = 1 : 2. AN and BM intersect at X.
(a) Find BM in terms of $\mathbf{a}$ and $\mathbf{b}$.
(b) Find OX in terms of $\mathbf{a}$ and $\mathbf{b}$.
The diagram is not drawn accurately.

(a) OM = $\frac{3}{4}\mathbf{a}$, so BM = $-\mathbf{b} + \frac{3}{4}\mathbf{a} = \frac{3}{4}\mathbf{a} - \mathbf{b}$

(b) X is on BM: BX = s BM, so OX = OB + s BM = $\frac{3}{4}s\mathbf{a} + (1 - s)\mathbf{b}$

X is on AN: OX = OA + t AN = OA + t(ON − OA) = $(1 - t)\mathbf{a} + \frac{1}{3}t\mathbf{b}$

Equate: $\frac{3}{4}s = 1 - t$ and $1 - s = \frac{1}{3}t$

Solving: $s = \frac{8}{9}$ and $t = \frac{1}{3}$

OX = $\frac{2}{3}\mathbf{a} + \frac{1}{9}\mathbf{b}$

Answer: (a) $\frac{3}{4}\mathbf{a} - \mathbf{b}$ (b) OX = $\frac{2}{3}\mathbf{a} + \frac{1}{9}\mathbf{b}$