Day 81 **S Tier 10-a-day: Model Solutions**

1 Averages with algebra

Here are five numbers: $x - 2$, x, $3x + 4$, $2x + 2$, $3x + 6$.
The mean of the five numbers is 8.
(a) Work out the value of x.
(b) Work out the range of the five numbers.

(a) Total = $(x - 2) + (x) + (3x + 4) + (2x + 2) + (3x + 6) = 8 \times 5 = 40$
$10x + 10 = 40$, so $10x = 30$ and $x = 3$
The numbers are 1, 3, 13, 8, 15
(b) Range = $15 - 1 = 14$
Answer: (a) x = 3 (b) 14

2 Percentage change in area and volume

The radius of a circle is increased by 50%.
Work out the percentage change in the area of the circle.

Radius multiplier = 1.5
Area multiplier = $1.5^2 = 2.25$
$2.25 - 1 = 1.25$, so a 125% increase
Answer: 125% increase

3 Solving quadratics by factorising

Solve $x^2 + 5x - 14 = 0$

Factorise: $(x - 2)(x + 7) = 0$
$x - 2 = 0$ or $x + 7 = 0$
$x = 2$ or $x = -7$
Answer: x = −7 or x = 2

4 Rational and irrational numbers

Here are six numbers.
$2\pi - \pi$, $3 + \sqrt{12}$, $\sqrt{8} \times \sqrt{2}$, $\sqrt{49}$, $\sqrt{50}$, $\sqrt{0.25}$
Write down the rational numbers.

Simplify each number:
$2\pi - \pi = \pi$; $3 + \sqrt{12} = 3 + 2\sqrt{3}$; $\sqrt{8} \times \sqrt{2} = \sqrt{16} = 4$
$\sqrt{49} = 7$; $\sqrt{50} = 5\sqrt{2}$; $\sqrt{0.25} = 0.5$
Answer: $\sqrt{8} \times \sqrt{2}$, $\sqrt{49}$, $\sqrt{0.25}$

5 Area of a triangle using sine

Triangle ABC has AB = 12 cm and BC = 6 cm.
The area of triangle ABC is 18 cm^2.
Angle ABC is obtuse.
Work out the size of angle ABC.
Do not use a calculator.

Area = $\frac{1}{2} \times AB \times BC \times \sin B$

$18 = \frac{1}{2} \times 12 \times 6 \times \sin B$

$\sin B = 18 \div 36 = \frac{1}{2}$

$\sin 30° = \frac{1}{2}$, and B is obtuse, so $B = 180° - 30° = 150°$

Answer: 150°

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6 Ratio and money transferred

Nadia and Callum share some money in the ratio 1 : 5.
Nadia gives Callum £5.
The ratio of their money is now 1 : 7.
How much money do they have altogether?

Let the amounts be 1k and 5k, so the total is 6k

Nadia now has 1k − 5, which is $\frac{1}{8}$ of the total

8(1k − 5) = 1 × 6k
8k − 40 = 6k, so 2k = 40 and k = 20
Total = 6 × 20 = £120
Answer: £120

7 Equations in disguise

Solve $x - 3\sqrt{x} + 2 = 0$

Let $u = \sqrt{x}$: $u^2 - 3u + 2 = 0$
Factorise: (u − 2)(u − 1) = 0, so u = 2 or u = 1
$\sqrt{x} = 2$ or $\sqrt{x} = 1$
x = 4 or x = 1
Answer: x = 1 or x = 4

8 Bearings with the sine and cosine rules

A walker leaves a camp P and travels 37 km on a bearing of 340° to Q. (Calculator)
It then travels 91 km on a bearing of 230° to R.
(a) Work out the distance PR.
(b) Work out the bearing of R from P.
Give distances to 1 decimal place and the bearing to the nearest degree.

(a) The bearing of P from Q is 340° − 180° = 160°
Angle PQR = 70° (the difference between the bearings 160° and 230°)
$PR^2 = 37^2 + 91^2 - 2 \times 37 \times 91 \times \cos 70° = 7346.8$, so PR = 85.7 km

(b) $\cos(QPR) = \frac{37^2 + PR^2 - 91^2}{2 \times 37 \times PR}$, so angle QPR = 86.1°

R is anticlockwise of the direction PQ, so the bearing is 340° − 86° = 254°
Answer: (a) 85.7 km (b) 254°

9 Growth and decay

A rare car is worth £800.
Its value increases by 8% each year.
After how many whole years will it first be worth more than £1,200? (Calculator)

Multiplier each year = 1.08
After 5 years: $£800 \times 1.08^5 = £1{,}175.46$
After 6 years: $£800 \times 1.08^6 = £1{,}269.50$
So the first time it is more than £1,200 is after 6 years
Answer: 6 years

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10 Tangent to a circle

A circle has equation $x^2 + y^2 = 289$.
The point P(8, −15) lies on the circle.
(a) Find the equation of the tangent to the circle at P. Give your answer in the form ax + by = c, where a, b and c are integers.
(b) The tangent meets the x-axis at the point T. Write down the coordinates of T.

(a) The gradient of the radius OP is $-\frac{15}{8}$

The tangent is perpendicular to the radius, so its gradient is $-1 \div (-\frac{15}{8}) = \frac{8}{15}$

$y - (-15) = \frac{8}{15}(x - 8)$

Multiply out and rearrange: $8x - 15y = 289$

(b) On the x-axis y = 0, so 8x = 289

$x = \frac{289}{8}$, so T is $(\frac{289}{8}, 0)$

Answer: (a) $8x - 15y = 289$ (b) T = $(\frac{289}{8}, 0)$