Day 80 **S Tier 10-a-day: Model Solutions**

1 Chords and Pythagoras in circles

AB is a chord of a circle, centre O. M is the midpoint of AB. (Calculator)
(a) Work out the length OM.
(b) Work out angle AOB, to 1 decimal place.
The diagram is not drawn accurately.

(a) AM = 14 ÷ 2 = 7 cm
$OM^2 = OA^2 - AM^2 = 25^2 - 7^2 = 576$
OM = 24 cm

(b) sin(angle AOM) = $\frac{7}{25}$
Angle AOM = 16.26°, so angle AOB = 2 × 16.26 = 32.5°
Answer: (a) 24 cm (b) 32.5°

2 Mean of a changing set

The mean of 11 numbers is 85.
One number is removed and the mean of the remaining 10 numbers is 87.5.
Work out the number that was removed.

Total of the 11 numbers = 11 × 85 = 935
Total of the 10 remaining = 10 × 87.5 = 875
Number removed = 935 − 875 = 60
Answer: 60

3 Completing the square

(a) Write $3x^2 - 12x + 21$ in the form $a(x + p)^2 + q$
(b) Write down the coordinates of the turning point of the curve $y = 3x^2 - 12x + 21$

(a) Take out the factor 3 from the x terms: $3(x^2 - 4x) + 21$
$x^2 - 4x = (x - 2)^2 - 4$, so $3x^2 - 12x + 21 = 3[(x - 2)^2 - 4] + 21$
$= 3(x - 2)^2 + 9$
(b) The minimum is when x − 2 = 0, so x = 2 and y = 9
Answer: (a) $3(x - 2)^2 + 9$ (b) (2, 9)

4 Percentages of percentages

In a group of visitors, 55% are adults and the rest are children.
20% of the adults and 50% of the children buy a souvenir.
What percentage of all the visitors buy a souvenir?

Take 100 visitors: 55 are adults and 45 are children
20% of 55 = 11 and 50% of 45 = 22.5
11 + 22.5 = 33.5 out of 100
Answer: 33.5%

5 Tree diagrams without replacement

A bag contains 3 orange sweets, 4 purple sweets and 3 pink sweets.
Two sweets are taken at random without replacement.
Work out the probability that (a) both are the same colour, (b) exactly one is purple.

(a) Add the ways: $\frac{3}{10} \times \frac{2}{9} + \frac{4}{10} \times \frac{3}{9} + \frac{3}{10} \times \frac{2}{9} = \frac{4}{15}$

(b) P(exactly one purple) = $\frac{4}{10} \times \frac{6}{9} + \frac{6}{10} \times \frac{4}{9} = \frac{8}{15}$

Answer: (a) $\frac{4}{15}$ (b) $\frac{8}{15}$

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6 Bounds in geometry

A right-angled triangle has hypotenuse 19.9 cm and one other side 6.6 cm, both correct to 1 decimal place. (Calculator)
Work out the lower bound of the length of the third side. Give your answer correct to 3 significant figures.

The third side is $\sqrt{\text{hypotenuse}^2 - \text{side}^2}$
For the lower bound, use the lower bound of the hypotenuse and the upper bound of the other side
$\sqrt{19.85^2 - 6.65^2} = 18.7$ cm (3 s.f.)
Answer: 18.7 cm

7 Inverse proportion in context

9 workers can build a wall in 15 days.
After 5 days, 4 of the workers leave. The rest carry on working at the same rate.
How many more days does it take to finish?

Total work = $9 \times 15 = 135$ worker-days
Work done = $9 \times 5 = 45$, so work left = 90
Workers left = $9 - 4 = 5$
Time = $90 \div 5 = 18$ days
Answer: 18 days

8 Equal roots and real roots

Show that the equation $x^2 + kx - 1 = 0$ has two distinct real solutions for all values of k.

$b^2 - 4ac = k^2 - 4 \times 1 \times (-1)$
$= k^2 + 4$
k^2 is never negative, so $k^2 + 4$ is always at least 4, which is greater than 0
So $b^2 - 4ac > 0$ for all k, and there are always two distinct real solutions
Answer: Shown

9 Perimeter and area of a running track

The diagram shows a running track made from a rectangle and two semicircles. (Calculator)
An athlete runs 5 complete laps around the track.
Work out the total distance run, to the nearest 10 metres.
The diagram is not drawn accurately.

Boundary of one lap = $2 \times 120 + 2 \times \pi \times 10 = 302.83$ m
5 laps = $5 \times 302.83 = 1514.2$ m
= 1510 m to the nearest 10 m
Answer: 1510 m

10 Index equations

Solve $4^{x+3} = 2^6$

Write both sides as powers of 2: $4 = 2^2$
Equate the powers: $2x + 6 = 6$
$2x = 0$, so $x = 0$
Answer: x = 0