Day 8 **S Tier 10-a-day: Model Solutions**

1 Tangents and circle theorems

PA and PB are tangents to a circle, centre O. C is a point on the circle. Angle APB = 54°.
(a) Work out the size of angle AOB.
(b) Work out the size of angle ACB.
The diagram is not drawn accurately.

(a) Angles OAP and OBP are 90° (a tangent meets a radius at 90°)
Angles in quadrilateral OAPB: 360 − 90 − 90 − 54 = 126°
(b) The angle at the circumference is half the angle at the centre: 126 ÷ 2 = 63°
Answer: (a) 126° (b) 63°

2 Laws of indices with algebra

Simplify fully $\frac{9x^4y \times 2xy}{2x^2y^3}$

Give your answer in the form kx^ay^b

Numbers: 9 × 2 ÷ 2 = 9
Powers of x: 4 + 1 − 2 = 3
Powers of y: 1 + 1 − 3 = −1
$= 9x^3y^{-1}$
Answer: $9x^3y^{-1}$

3 Angles with algebra

In triangle PQR, side QR is extended to S.
Angle QPR = (2x)°
Angle PQR = (3x + 30)°
Angle PRS = (4x + 57)°
(a) Work out the value of x.
(b) Work out the size of angle PRQ.

(a) The exterior angle of a triangle equals the sum of the two opposite interior angles
4x + 57 = 5x + 30
x = 27
(b) Angle PRS = 165° so angle PRQ = 180° − 165° = 15°
Answer: (a) x = 27 (b) 15°

4 Tests of divisibility

The 4-digit number 656k is a multiple of 9.
k is a single digit.
Find all the possible values of k.

A number is a multiple of 9 if its digit sum is a multiple of 9
Digit sum = 6 + 5 + 6 + k = 17 + k
This must be a multiple of 9, so k = 1
Answer: k = 1

5 Speed-time graphs

The graph shows the speed of a cyclist during 12 seconds.
(a) Work out the acceleration during the first 3 seconds.
(b) Work out the average speed over the whole 12 seconds.

(a) Acceleration = change in speed ÷ time = (24 − 0) ÷ 3 = 8 m/s^2
(b) Distance = area under the graph
Trapezium: $\frac{0 + 24}{2} \times 3 = 36$; rectangle: 24 × 4 = 96; triangle: $\frac{1}{2} \times 5 \times 24 = 60$
Total = 192 m, so average speed = 192 ÷ 12 = 16 m/s
Answer: (a) 8 m/s^2 (b) 16 m/s

Day 8 **S Tier 10-a-day: Model Solutions**

6 Expanding brackets with surds

Expand and simplify $(3 - \sqrt{7})(2 + 2\sqrt{7})$

Give your answer in the form $p + q\sqrt{7}$

$(3 - \sqrt{7})(2 + 2\sqrt{7}) = 6 + 6\sqrt{7} - 2\sqrt{7} - 14$

$= -8 + 4\sqrt{7}$

Answer: $-8 + 4\sqrt{7}$

7 Sample space and conditional probability

A fair 6-sided dice and a spinner numbered 1 to 5 are each thrown once. The first score and the second score are recorded.

(a) Work out the probability that the product is even.

(b) Given that the first number is greater than the second, work out the probability that the product is even.

(a) There are $6 \times 5 = 30$ equally likely outcomes, and 21 of them satisfy: the product is even

Probability = $\frac{7}{10}$

(b) 15 outcomes satisfy: the first number is greater than the second. Of these, 12 also satisfy: the product is even

Probability = $\frac{4}{5}$

Answer: (a) $\frac{7}{10}$ (b) $\frac{4}{5}$

8 Prime factors and indices

Write $8^3 \times 12^3$ as a product of powers of prime numbers.

$8 = 2^3$, $12 = 2^2 \times 3$

$8^3 = 2^9$

$12^3 = 2^6 \times 3^3$

Add the powers of each prime: $2^{15} \times 3^3$

Answer: $2^{15} \times 3^3$

9 Angles of elevation

A point A is on level ground in line with the base of a lighthouse. (Calculator)

The angle of elevation of the top of the lighthouse from A is 37°.

B is a point on the same line, 30 m further from the lighthouse than A.

The angle of elevation of the top of the lighthouse from B is 21°.

Work out the height of the lighthouse. Give your answer to 1 decimal place.

Let the height be h and the distance from A to the base be x. $\tan 37° = \frac{h}{x}$, so $x = h \div \tan 37°$

From B: $\tan 21° = \frac{h}{x + 30}$, so $x + 30 = h \div \tan 21°$

Subtract: $30 = h \div \tan 21° - h \div \tan 37° = h(2.6051 - 1.3270)$

$h = 30 \div 1.2780 = 23.5$ m (1 d.p.)

Answer: 23.5 m

10 Tangency conditions

The line $y = 3x + k$ is a tangent to the circle $x^2 + y^2 = 15$.

Find the possible values of k.

Give any surds in their simplest form.

Substitute $y = 3x + k$: $x^2 + (3x + k)^2 = 15$

Expand: $10x^2 + 6kx + k^2 - 15 = 0$

A tangent touches at one point, so $b^2 - 4ac = 0$: $(6k)^2 - 4 \times 10 \times (k^2 - 15) = 0$

$-4k^2 + 600 = 0$, so $k^2 = 150$

$k = \pm 5\sqrt{6}$

Answer: $k = \pm 5\sqrt{6}$