Day 79 **S Tier 10-a-day: Model Solutions**

1 Mean of a changing set

The mean of 5 numbers is 87.
One number is removed and the mean of the remaining 4 numbers is 105.75.
Work out the number that was removed.

Total of the 5 numbers = 5 × 87 = 435
Total of the 4 remaining = 4 × 105.75 = 423
Number removed = 435 − 423 = 12
Answer: 12

2 Perimeter and area of a running track

The diagram shows a running track made from a rectangle and two semicircles.
(a) Work out the total length of the boundary of the track.
(b) Work out the area enclosed by the track. Give your answers in terms of π.
The diagram is not drawn accurately.

(a) The two semicircles make one full circle of radius 10: circumference = 2 × π × 10 = 20π
The two straight sides: 2 × 100 = 200
Boundary = (200 + 20π) m
(b) The rectangle is 100 m by 2 × 10 = 20 m, so its area is 100 × 20 = 2000. Circle: π × 10^2 = 100π
Area = (2000 + 100π) m^2
Answer: (a) (200 + 20π) m (b) (2000 + 100π) m^2

3 Distance-time graphs

The graph shows Fatima's cycle ride to a café and back.
(a) Work out Fatima's speed on the way back.
(b) Work out Fatima's average speed for the whole ride, including the stop.

(a) Return: 15 km in 0.5 hours, so speed = 15 ÷ 0.5 = 30 km/h
(b) Total distance = 2 × 15 = 30 km, total time = 2 hours
Average speed = 30 ÷ 2 = 15 km/h
Answer: (a) 30 km/h (b) 15 km/h

4 Arc length and sector area

A sector of a circle has angle 120° and area 75π cm^2.
(a) Work out the radius of the circle.
(b) Work out the length of the arc of the sector. Give your answer to (b) in terms of π.

(a) Area = $\frac{120}{360}$ × π × r^2, so $\frac{120}{360}$ × r^2 = 75

$r^2 = 75 \times \frac{360}{120} = 225$

r = 15 cm

(b) Arc length = $\frac{120}{360}$ × 2 × π × 15 = 10π cm
Answer: (a) 15 cm (b) 10π cm

5 Upper and lower bounds

a = 5.9 and b = 3.7,
both correct to 1 decimal place.
Work out the upper bound of a − b.

Each value is rounded, so it can be up to 0.05 above or below
5.85 ≤ a < 5.95; 3.65 ≤ b < 3.75
Biggest answer: biggest a minus smallest b = 5.95 − 3.65 = 2.3
Answer: 2.3

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6 Circle theorems: angle at the centre

A, B and C are points on a circle, centre O. Angle OAB = 66°.
(a) Work out the size of angle AOB.
(b) Work out the size of angle ACB.
The diagram is not drawn accurately.

(a) OA = OB (radii), so triangle OAB is isosceles and angle OBA = 66°
Angle AOB = 180 − 2 × 66 = 48°
(b) The angle at the circumference is half the angle at the centre: 48 ÷ 2 = 24°
Answer: (a) 48° (b) 24°

7 Winning a match

Tariq and Grace play a match. The first to win 2 tennis sets wins the match.
The probability that Tariq wins any one of the tennis sets is $\frac{7}{10}$.
Work out the probability that the match needs a third one to decide it.

A third one is needed when each player wins one of the first two
P(Tariq wins then loses) = $\frac{7}{10} \times \frac{3}{10} = \frac{21}{100}$
P(Tariq loses then wins) = $\frac{3}{10} \times \frac{7}{10} = \frac{21}{100}$
Total = $2 \times \frac{21}{100} = \frac{21}{50}$
Answer: $\mathbf{\frac{21}{50}}$

8 Algebraic fractions: multiplying and dividing

Simplify fully $\frac{x^2 - 6x + 8}{x^2 + 6x} \times \frac{x^2 - x}{x - 4}$

Factorise: $\frac{(x - 4)(x - 2)}{x(x + 6)} \times \frac{x(x - 1)}{(x - 4)}$
Cancel $(x - 4)$ and x
$= \frac{(x - 2)(x - 1)}{(x + 6)}$
Answer: $\mathbf{\frac{(x - 2)(x - 1)}{(x + 6)}}$

9 Percentages with an unknown rate

The price of a car is increased by x%.
The new price is then decreased by x%.
The final price is 25% less than the original price.
Work out the value of x.

Let the original price be 1. Multipliers: $(1 + \frac{x}{100})$ and $(1 - \frac{x}{100})$
$(1 + \frac{x}{100})(1 - \frac{x}{100}) = 1 - \frac{x^2}{10000}$
Final price is 25% less, so the multiplier is 0.75: $1 - \frac{x^2}{10000} = 0.75$
$\frac{x^2}{10000} = 0.25$, so $x^2 = 2500$ and $x = 50$
Answer: x = 50

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10 Iteration: rearranging equations

(a) Show that the equation $x^3 - 2x - 30 = 0$ has a solution between $x = 3$ and $x = 4$.

(b) Show that the equation can be rearranged to give $x = \sqrt[3]{2x + 30}$

(a) Let $f(x) = x^3 - 2x - 30$. $f(3) = 3^3 - 2 \times 3 - 30 = -9$ and $f(4) = 4^3 - 2 \times 4 - 30 = 26$

f(3) is negative and f(4) is positive. There is a change of sign, so there is a solution between 3 and 4

(b) Rearrange $x^3 - 2x - 30 = 0$: $x^3 = 2x + 30$

Take the cube root of both sides: $x = \sqrt[3]{2x + 30}$

Answer: Shown