**Day 77** **S Tier 10-a-day: Model Solutions**

## 1 Angles with algebra

ABC is an isosceles triangle with AB = AC.
Angle BAC = (2x − 10)°
Angle ABC = (3x + 35)°
(a) Work out the value of x.
(b) Work out the size of angle BAC.

(a) Angles ABC and ACB are equal, and the angles in a triangle add up to 180°
(2x − 10) + 2(3x + 35) = 180
8x + 60 = 180, so 8x = 120
x = 15
(b) Angle BAC = 2 × 15 − 10 = 20°
**Answer: (a) x = 15 (b) 20°**

## 2 Standard form

Work out $(7.95 \times 10^4) \div (1.5 \times 10^2)$
Give your answer in standard form.

7.95 ÷ 1.5 = 5.3
$10^4 \div 10^2 = 10^2$
$= 5.3 \times 10^2$
**Answer: $5.3 \times 10^2$**

## 3 Weighted percentages

The ratio of juniors to seniors in a school is 3 : 2.
45% of the juniors and 70% of the seniors play a sport.
What percentage of all of these students play a sport?

Imagine 100 students in total: 60 juniors and 40 seniors
45% of 60 = 27 and 70% of 40 = 28
Together: 55 out of 100
55 ÷ 100 × 100 = 55%
**Answer: 55%**

## 4 Sine rule

The diagram shows triangle ABC. (Calculator)
Work out the length of AC. Give your answer correct to 1 decimal place.
The diagram is not drawn accurately.

Use the sine rule: $\frac{AC}{\sin B} = \frac{BC}{\sin A}$

$\frac{x}{\sin 73°} = \frac{11}{\sin 37°}$

$x = \frac{11 \times \sin 73°}{\sin 37°} = 17.479$

x = 17.5 cm (1 d.p.)
**Answer: AC = 17.5 cm**

## 5 Cumulative frequency graphs

The cumulative frequency graph gives information about the masses of 80 apples picked from a tree.
(a) Find an estimate for the median and the interquartile range.
(b) Estimate how many of the 80 had a mass greater than 35.

(a) Median at cumulative frequency 40, so median = 30
Lower quartile at 20, so LQ = 20. Upper quartile at 60, so UQ = 40
Interquartile range = 40 − 20 = 20
(b) At 35, cumulative frequency = 50
Number greater = 80 − 50 = 30
**Answer: (a) median 30, IQR 20 (b) 30**

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## 6 Index equations

Solve $8^{2x} = 16^{x+5}$

Write both sides as powers of 2: $8 = 2^3$ and $16 = 2^4$
Equate the powers: $6x = 4x + 20$
$2x = 20$, so $x = 10$
**Answer: x = 10**

## 7 Iteration: rearranging equations

(a) Show that the equation $x^2 - 8x - 22 = 0$ can be rearranged to give $x = 8 + \frac{22}{x}$ (Calculator)

(b) Use the iteration formula $x_{n+1} = 8 + \frac{22}{x_n}$ with $x_0 = 10$ to work out $x_1$ and $x_2$. Give each answer correct to 3 decimal places.

(a) Rearrange $x^2 - 8x - 22 = 0$: $x^2 - 8x = 22$
Factorise: $x(x - 8) = 22$
Divide by x: $x - 8 = \frac{22}{x}$, so $x = 8 + \frac{22}{x}$

(b) $x_1 = 8 + \frac{22}{10} = 10.20000...$

$x_2 = 8 + \frac{22}{x_1} = 10.15686...$

$x_1 = 10.200$ and $x_2 = 10.157$ (3 d.p.)
**Answer: (b) $x_1 = 10.200$, $x_2 = 10.157$**

## 8 Perimeter and area of a running track

The diagram shows a running track made from a rectangle and two semicircles.
(a) Work out the total length of the boundary of the track.
(b) Work out the area enclosed by the track. Give your answers in terms of $\pi$.
The diagram is not drawn accurately.

(a) The two semicircles make one full circle of radius 30: circumference $= 2 \times \pi \times 30 = 60\pi$
The two straight sides: $2 \times 40 = 80$
Boundary = $(80 + 60\pi)$ m
(b) The rectangle is 40 m by $2 \times 30 = 60$ m, so its area is $40 \times 60 = 2400$. Circle: $\pi \times 30^2 = 900\pi$
Area = $(2400 + 900\pi)$ m$^2$
**Answer: (a) $(80 + 60\pi)$ m (b) $(2400 + 900\pi)$ m$^2$**

## 9 Tangency conditions

The line $y = kx - 3$ is a tangent to the curve $y = x^2 + 5$.
Find the possible values of k.
Give any surds in their simplest form.

Equate: $x^2 + 5 = kx - 3$
Rearrange: $x^2 - kx + 8 = 0$
One point of contact, so $b^2 - 4ac = 0$: $k^2 - 4 \times 8 = 0$
$k^2 = 32$
$k = \pm\sqrt{32} = \pm 4\sqrt{2}$
**Answer: $k = \pm 4\sqrt{2}$**

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## 10 Sample space and conditional probability

A spinner numbered 1 to 4 and a spinner numbered 1 to 5 are each thrown once. The first score and the second score are recorded.

(a) Work out the probability that the product is even.

(b) Given that the first number is greater than the second, work out the probability that the product is even.

(a) There are 4 × 5 = 20 equally likely outcomes, and 14 of them satisfy: the product is even

Probability = $\frac{7}{10}$

(b) 6 outcomes satisfy: the first number is greater than the second. Of these, 5 also satisfy: the product is even

Probability = $\frac{5}{6}$

**Answer: (a) $\frac{7}{10}$ (b) $\frac{5}{6}$**