Day 76 **S Tier 10-a-day: Model Solutions**

1 Relative speed

Oscar sets off at 10:00 and cycles at 10 km/h along a path.
Carlos sets off from the same place at 11:00 and cycles at 34 km/h along the same path.
At what time does Carlos catch up with Oscar?

When Carlos starts, Oscar is $1 \times 10 = 10$ km ahead
Carlos gains $34 - 10 = 24$ km every hour

Time to catch up = $10 \div 24 = \frac{5}{12}$ hours = 25 minutes

11:00 + 25 minutes = 11:25
Answer: 11:25

2 Circle theorems: angle at the centre

A, B and C are points on a circle, centre O. Angle OAB = 50°.
(a) Work out the size of angle AOB.
(b) Work out the size of angle ACB.
The diagram is not drawn accurately.

(a) OA = OB (radii), so triangle OAB is isosceles and angle OBA = 50°
Angle AOB = $180 - 2 \times 50 = 80°$
(b) The angle at the circumference is half the angle at the centre: $80 \div 2 = 40°$
Answer: (a) 80° (b) 40°

3 Simplifying surds

Write $\sqrt{18} + \sqrt{32} + \sqrt{2}$ in the form $a\sqrt{2}$
where a is an integer.

$\sqrt{18} = 3\sqrt{2}$; $\sqrt{32} = 4\sqrt{2}$
$(3 + 4 + 1)\sqrt{2} = 8\sqrt{2}$
Answer: $8\sqrt{2}$

4 Quadratic inequalities

Solve $x^2 \leq 25$

Rearrange: $x^2 - 25 \leq 0$
Solve $x^2 - 25 = 0$: $(x + 5)(x - 5) = 0$, so $x = -5$ or $x = 5$
The graph is a U shape crossing the x-axis at -5 and 5; it is below the axis between the roots
$-5 \leq x \leq 5$
Answer: $-5 \leq x \leq 5$

5 Successive percentage change

The price of a laptop is increased by 60%.
The new amount is then decreased by 25%.
Work out the overall percentage change.

Multipliers: 1.6 and 0.75
$1.6 \times 0.75 = 1.2$
1.2 is an increase of 20%
Answer: 20% increase

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6 Areas of similar triangles

In the diagram, DE is parallel to BC. AD = 6 cm, DB = 15 cm and the area of triangle ADE is 32 cm^2.
Work out the area of the trapezium DBCE.
The diagram is not drawn accurately.

Triangles ADE and ABC are similar. AB = 6 + 15 = 21 cm

Length scale factor = $21 \div 6 = \frac{7}{2}$

Area scale factor = $(\frac{7}{2})^2 = \frac{49}{4}$

Area of triangle ABC = $32 \times \frac{49}{4} = 392$ cm^2

Trapezium = $392 - 32 = 360$ cm^2

Answer: 360 cm^2

7 Prime factors and indices

Write $24^2 \times 20^3$ as a product of powers of prime numbers.

$24 = 2^3 \times 3$, $20 = 2^2 \times 5$

$24^2 = 2^6 \times 3^2$

$20^3 = 2^6 \times 5^3$

Add the powers of each prime: $2^{12} \times 3^2 \times 5^3$

Answer: $2^{12} \times 3^2 \times 5^3$

8 Vector proof: collinear points

OA = $\mathbf{a}$ and OB = $\mathbf{b}$.
OP = $\mathbf{a} + 3\mathbf{b}$, OQ = $-7\mathbf{a} + 13\mathbf{b}$ and OR = $-23\mathbf{a} + 33\mathbf{b}$.
(a) Show that P, Q and R lie on a straight line.
(b) Work out the ratio PQ : QR in its simplest form.

(a) PQ = OQ − OP = $-8\mathbf{a} + 10\mathbf{b}$
QR = OR − OQ = $-16\mathbf{a} + 20\mathbf{b}$
QR = 2 PQ, so PQ and QR are parallel, and they share the point Q
So P, Q and R lie on a straight line
(b) PQ : QR = 2 : 4 = 1 : 2
Answer: (b) 1 : 2

9 Squares, cubes and prime factors

Find the smallest positive integer k such that 40 × k is a square number.

$40 = 2^3 \times 5$
For a square number every power must be a multiple of 2
Multiply by 2×5 to get $2^4 \times 5^2$
k = 10
Answer: k = 10

10 Tangency conditions

The line $y = 4x + k$ is a tangent to the curve $y = x^2 - 4x + 4$.
Find the value of k.

Equate: $x^2 - 4x + 4 = 4x + k$
Rearrange: $x^2 - 8x + 4 - k = 0$
A tangent touches at one point, so $b^2 - 4ac = 0$: $64 - 4 \times 1 \times (4 - k) = 0$
$4k + 48 = 0$
$k = -12$
Answer: k = −12