Day 75 **S Tier 10-a-day: Model Solutions**

1 Capture and recapture

80 deer in a forest are caught, marked and put back.
Later, a second sample of 30 deer is caught. 10 of these are marked.
(a) Work out an estimate for the number of deer in a forest.
(b) State one assumption you have made.

(a) The proportion marked in the second sample is 10 out of 30

This should match the proportion marked in the whole population: $\frac{10}{30} = \frac{80}{N}$

N = 80 × 30 ÷ 10 = 240

(b) No deer are born, die or leave between the samples, and the deer mix evenly

Answer: (a) 240 (b) The population stays the same between samples

2 Vectors in a triangle

OAB is a triangle. OA = $\mathbf{a}$ and OB = $\mathbf{b}$.
M is on OA with OM : MA = 1 : 3. N is on AB with AN : NB = 1 : 3.
(a) Find ON in terms of $\mathbf{a}$ and $\mathbf{b}$.
(b) Find MN in terms of $\mathbf{a}$ and $\mathbf{b}$.
The diagram is not drawn accurately.

(a) AB = $-\mathbf{a} + \mathbf{b}$, so AN = $\frac{1}{4}(\mathbf{b} - \mathbf{a})$

ON = OA + AN = $\mathbf{a} + \frac{1}{4}(\mathbf{b} - \mathbf{a}) = \frac{3}{4}\mathbf{a} + \frac{1}{4}\mathbf{b}$

(b) OM = $\frac{1}{4}\mathbf{a}$

MN = ON − OM = $\frac{3}{4}\mathbf{a} + \frac{1}{4}\mathbf{b} - \frac{1}{4}\mathbf{a} = \frac{1}{2}\mathbf{a} + \frac{1}{4}\mathbf{b}$

Answer: (a) $\frac{3}{4}\mathbf{a} + \frac{1}{4}\mathbf{b}$ (b) $\frac{1}{2}\mathbf{a} + \frac{1}{4}\mathbf{b}$

3 Inequalities

(a) Solve $3 - 4x \geq -13$
(b) n is an integer. Write down all the values of n that satisfy
$-1 \leq 2n - 5 \leq 11$

(a) Subtract 3: $-4x \geq -16$
Divide by −4 and reverse the inequality sign: $x \leq 4$
(b) Add 5: $4 \leq 2n \leq 16$
Divide by 2: $2 \leq n \leq 8$
Integers: 2, 3, 4, 5, 6, 7, 8
Answer: (a) $x \leq 4$ (b) 2, 3, 4, 5, 6, 7, 8

4 Reverse percentages

The price of a sofa is increased by 25%.
Later the new price is reduced by 10% in a sale.
The sale price is £540.
Work out the price before the increase. (Calculator)

Multipliers: 1.25 and 0.9
Overall multiplier = 1.25 × 0.9 = 1.125
Original × 1.125 = £540
Original = £540 ÷ 1.125 = £480
Answer: £480

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5 Income tax

In one country, income tax is worked out like this:
The first £12,570 of income is not taxed.
The next £30,000 is taxed at 20%.
All income above that is taxed at 40%.
Ivan earns £72,570 in a year.
Work out how much income tax Ivan pays.

Taxed at 20%: £30,000
20% of £30,000 = £6,000
Taxed at 40%: £72,570 − £12,570 − £30,000 = £30,000
40% of £30,000 = £12,000
Total tax = £6,000 + £12,000 = £18,000
Answer: £18,000

6 Enlargement

Triangle T has vertices A(3, 0), B(3, −1), C(5, −1).
T is enlarged with scale factor −3 and centre (4, 0).
(a) Write down the coordinates of the image of A.
(b) The area of T is 24 square units. Work out the area of the image of T.

(a) Vector from the centre to A = (3 − 4, 0 − 0) = (−1, 0)
Multiply by −3: (3, 0)
Image of A = centre + this vector = (7, 0)
(b) Area scale factor = $(-3)^2 = 9$
Area = 24 × 9 = 216 square units
Answer: (a) (7, 0) (b) 216

7 Algebraic fractions: adding and subtracting

Show that $\frac{4}{3x+1} - \frac{x+1}{x-5}$ can be written as $\frac{-3x^2-21}{(3x+1)(x-5)}$

The common denominator is (3x + 1)(x − 5)

$$\frac{4}{3x+1} - \frac{x+1}{x-5} = \frac{4(x-5)-(x+1)(3x+1)}{(3x+1)(x-5)}$$

Expand the numerator: $-3x^2 - 21$

$$\frac{-3x^2-21}{(3x+1)(x-5)}$$

Answer: $\mathbf{\frac{-3x^2-21}{(3x+1)(x-5)}}$

8 Estimating the median from grouped data

The table gives information about 104 lengths.
Work out an estimate for the median.
Assume the values in each class are evenly spread.

Cumulative frequencies: 23, 41, 66, 80, 104
The median is the value at position 104 ÷ 2 = 52
It is in the class 30 to 40, with 41 values before it, so we need 11 more of the 25 values in this class

Median ≈ $30 + \frac{11}{25} \times 10 = 34.4$

Answer: 34.4

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9 Bounds of roots

The volume of a cube is 700 cm^3, correct to the nearest cm^3.
Work out the upper bound of the edge length of the cube.
Give your answer correct to 3 significant figures. (Calculator)

$699.5 \leq$ volume < 700.5

Upper bound of edge = $\sqrt[3]{700.5}$
= 8.88115... = 8.88 (3 s.f.)
Answer: 8.88 cm

10 Perpendicular bisector

A is the point $(5, -3)$ and B is the point $(9, -5)$.
Find an equation of the perpendicular bisector of AB.
Give your answer in the form $y = mx + c$.

Midpoint of AB = $(\frac{5+9}{2}, \frac{-3+(-5)}{2}) = (7, -4)$

Gradient of AB = $\frac{-5-(-3)}{9-5} = -\frac{1}{2}$

The perpendicular gradient is $-1 \div (-\frac{1}{2}) = 2$

Using the midpoint: $y + 4 = 2(x - 7)$
$y = 2x - 18$
Answer: $y = 2x - 18$