Day 74 **S Tier 10-a-day: Model Solutions**

1 Angle in a semicircle

AB is a diameter of a circle, centre O. C is a point on the circle. Angle BAC = 47°.
(a) Work out the size of angle ACB.
(b) Work out the size of angle COB.
The diagram is not drawn accurately.

(a) The angle in a semicircle is 90°, so angle ACB = 90°
(b) OA = OC (radii), so triangle OAC is isosceles and angle OCA = 47°
Angle COB is an exterior angle of triangle OAC: 47 + 47 = 94°
(or: the angle at the centre is twice the angle at the circumference)
Answer: (a) 90° (b) 94°

2 Quadratic formula and surds

Solve $x^2 - 6x - 4 = 0$
Give your answers in the form $a \pm \sqrt{b}$

Complete the square: $(x - 3)^2 - 9 - 4 = 0$
$(x - 3)^2 = 13$
$x - 3 = \pm\sqrt{13}$
$x = 3 \pm \sqrt{13}$
Answer: $x = 3 \pm \sqrt{13}$

3 Error intervals

The volume of juice is 630 litres, correct to the nearest 10.
Write down the error interval for the volume of juice, using v.

The value has been rounded to the nearest 10
Half of this is 5, so go 5 below and 5 above
630 − 5 = 625 and 630 + 5 = 635
$625 \leq v < 635$
Answer: $625 \leq v < 635$

4 Quadratic graphs

The curve $y = x^2 - 18x + 65$ crosses the x-axis at the points A and B.
(a) Find the coordinates of A and B.
(b) Find the coordinates of the minimum point of the curve.

(a) Put y = 0: $x^2 - 18x + 65 = 0$, so $(x - 5)(x - 13) = 0$
$x - 5 = 0$ or $x - 13 = 0$, so $x = 5$ or $x = 13$
(b) The turning point is halfway between A and B: $x = (5 + 13) \div 2 = 9$
When x = 9: $y = 9^2 - 18 \times 9 + 65 = -16$
Answer: (a) A(5, 0), B(13, 0) (b) (9, −16)

5 Fractions of fractions

Uma spent $\frac{2}{5}$ of her money on a phone.

She then spent $\frac{1}{7}$ of the money that was left on a case.

A case cost £303.
How much money did Uma have to start with?

The second amount is $\frac{1}{7}$ of $\frac{3}{5}$, which is $\frac{3}{35}$ of the total

$\frac{3}{35}$ of the total is £303

Total = £303 ÷ $\frac{3}{35}$ = £3,535

Answer: £3,535

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6 Trigonometric graphs

The diagram shows the graph of $y = a\cos x° + c$ for $0 \le x \le 360$.
Some points on the graph are labelled.
Find the value of a and the value of c.

At $x = 0$, $\cos x° = 1$, so $a + c = -4$
At $x = 180$, $\cos x° = -1$, so $-a + c = -8$
Add: $2c = -12$, so $c = -6$
Subtract: $2a = 4$, so $a = 2$
Answer: a = 2, c = −6

7 Cumulative frequency graphs

The cumulative frequency graph gives information about the marks of 80 students in a test.
(a) Find an estimate for the median and the interquartile range.
(b) Estimate how many of the 80 had a mark greater than 45.

(a) Median at cumulative frequency 40, so median = 40
Lower quartile at 20, so LQ = 30. Upper quartile at 60, so UQ = 50
Interquartile range = 50 − 30 = 20
(b) At 45, cumulative frequency = 50
Number greater = 80 − 50 = 30
Answer: (a) median 40, IQR 20 (b) 30

8 Cone made from a sector

A sector of a circle has radius 10 cm and angle 180°.
The sector is made into a cone by joining the two straight edges together.
(a) Work out the radius of the base of the cone.
(b) Work out the exact volume of the cone. Give your answer in terms of π, as a surd if necessary.

(a) Arc length of the sector = $\frac{180}{360} \times 2\pi \times 10 = 10\pi$ cm
This is the circumference of the base: $2\pi r = 10\pi$, so $r = 5$ cm
(b) Slant height = 10 cm, so height$^2 = 10^2 - 5^2 = 75$, height $= 5\sqrt{3}$ cm
Volume $= \frac{1}{3} \times \pi \times 5^2 \times 5\sqrt{3} = \frac{125\sqrt{3}}{3}\pi$ cm^3
Answer: (a) 5 cm (b) $\frac{125\sqrt{3}}{3}\pi$ cm^3

9 Exchange rates

Amir changes £800 into dollars at £1 = $1.15.
Amir spends $700 and then changes the rest back into pounds at £1 = $1.25.
How many pounds does Amir get back?

£800 × 1.15 = $920
920 − 700 = $220
220 ÷ 1.25 = £176
Answer: £176

10 Growth and decay

A rare car is worth £8,000.
Its value increases by 12% each year.
After how many whole years will it first be worth more than £20,000? (Calculator)

Multiplier each year = 1.12
After 8 years: £8,000 × 1.12^8 = £19,807.71
After 9 years: £8,000 × 1.12^9 = £22,184.63
So the first time it is more than £20,000 is after 9 years
Answer: 9 years