Day 73 **S Tier 10-a-day: Model Solutions**

1 Circle theorems: angle at the centre

A, B and C are points on a circle, centre O. Angle ACB = 24°.
(a) Work out the size of angle AOB.
(b) Work out the size of angle OAB.
The diagram is not drawn accurately.

(a) The angle at the centre is twice the angle at the circumference: 2 × 24 = 48°
(b) OA = OB (radii), so triangle OAB is isosceles
Angle OAB = (180 − 48) ÷ 2 = 66°
Answer: (a) 48° (b) 66°

2 Completing the square

(a) Write $4x^2 + 32x + 59$ in the form $a(x + p)^2 + q$
(b) Write down the coordinates of the turning point of the curve $y = 4x^2 + 32x + 59$

(a) Take out the factor 4 from the x terms: $4(x^2 + 8x) + 59$
$x^2 + 8x = (x + 4)^2 - 16$, so $4x^2 + 32x + 59 = 4[(x + 4)^2 - 16] + 59$
$= 4(x + 4)^2 - 5$
(b) The minimum is when $x + 4 = 0$, so $x = -4$ and $y = -5$
Answer: (a) $4(x + 4)^2 - 5$ (b) $(-4, -5)$

3 Rational and irrational numbers

Here are six numbers.

$\frac{\sqrt{20}}{\sqrt{5}}$, $(1 + \sqrt{2})^2$, $\sqrt{0.25}$, $0.\dot{4}$, $\sqrt{50}$, $\sqrt{8} \times \sqrt{2}$

Write down the irrational numbers.

Simplify each number:

$\frac{\sqrt{20}}{\sqrt{5}} = \sqrt{4} = 2$; $(1 + \sqrt{2})^2 = 3 + 2\sqrt{2}$; $\sqrt{0.25} = 0.5$

$0.\dot{4} = \frac{4}{9}$; $\sqrt{50} = 5\sqrt{2}$; $\sqrt{8} \times \sqrt{2} = \sqrt{16} = 4$

Answer: $(1 + \sqrt{2})^2$, $\sqrt{50}$

4 Fractions removed from a ratio

The ratio of red to blue marbles in a jar is 3 : 2.
Then $\frac{2}{3}$ of the red and $\frac{1}{6}$ of the blue are taken out.
Work out the ratio of red to blue now.

Left: $\frac{1}{3} \times 3 = 1$ of the red part, $\frac{5}{6} \times 2 = \frac{5}{3}$ of the blue part

$1 : \frac{5}{3}$

Multiply by 3: 3 : 5
Answer: 3 : 5

5 Equations with indices

Solve $5^x = \frac{1}{125}$

Write the right-hand side as a power of 5: $\frac{1}{125} = 5^{-3}$

Equate the powers: $x = -3$
$x = -3$
Answer: $x = -3$

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6 Similar shapes

Two bottles are mathematically similar.
The heights of the bottles are 6 cm and 15 cm.
The smaller one has capacity 80 ml.
Work out the capacity of the larger one.

Scale factor of lengths = $15 \div 6 = \frac{5}{2}$

Scale factor of volumes = $(\frac{5}{2})^3 = \frac{125}{8}$

$80 \times \frac{125}{8} = 1250$ ml

Answer: 1250 ml

7 Moving averages

The table shows the monthly sales (thousands) over six periods.
The 3-point moving average centred on Mar is 24.
(a) Work out the value of x.
(b) Work out the 3-point moving average centred on Apr.

(a) $(33 + x + 26) \div 3 = 24$
$33 + x + 26 = 72$, so $x = 13$
(b) $(13 + 26 + 36) \div 3 = 75 \div 3 = 25$
Answer: (a) x = 13 (b) 25

8 3D problems: pyramids

VABCD is a pyramid on a square base. V is directly above the centre M.
(a) Work out the height VM of the pyramid.
(b) Work out the volume of the pyramid.
The diagram is not drawn accurately.

(a) $AC^2 = 4^2 + 4^2 = 32$, so $AM^2 = (AC \div 2)^2 = 8$
$VM^2 = VA^2 - AM^2 = 3^2 - 8 = 1$
$VM = 1$ cm
(b) Volume $= \frac{1}{3} \times 4^2 \times 1 = \frac{16}{3}$ cm^3

Answer: (a) 1 cm (b) $\frac{16}{3}$ cm^3

9 Percentages of percentages

In a group of members, 70% are juniors and the rest are seniors.
30% of the juniors and 35% of the seniors play chess.
What percentage of all the members play chess?

Take 100 members: 70 are juniors and 30 are seniors
30% of 70 = 21 and 35% of 30 = 10.5
21 + 10.5 = 31.5 out of 100
Answer: 31.5%

10 Equal roots and real roots

The equation $kx^2 - 3x + 3 = 0$ has equal roots.
Find the value of k.

Equal roots means $b^2 - 4ac = 0$
$(-3)^2 - 4 \times k \times 3 = 0$, so $9 = 12k$
$k = \frac{3}{4}$

Answer: k = $\frac{3}{4}$