Day 72 **S Tier 10-a-day: Model Solutions**

1 Perpendicular bisector

A is the point (−2, −3) and B is the point (6, −9).
Find an equation of the perpendicular bisector of AB.
Give your answer in the form ax + by = c, where a, b and c are integers.

Midpoint of AB = $(\frac{-2+6}{2}, \frac{-3+(-9)}{2}) = (2, -6)$

Gradient of AB = $\frac{-9-(-3)}{6-(-2)} = -\frac{3}{4}$

The perpendicular gradient is $-1 \div (-\frac{3}{4}) = \frac{4}{3}$

Using the midpoint: $y + 6 = \frac{4}{3}(x - 2)$

Multiply by 3 and rearrange: $-4x + 3y = -26$
Multiply by −1: $4x - 3y = 26$
Answer: $4x - 3y = 26$

2 Pressure

A stone block in the shape of a cuboid weighs 420 N.
Its smallest face has area 300 cm^2. Its largest face has area 600 cm^2.
Pressure = force ÷ area
Work out (a) the greatest pressure and (b) the least pressure the stone block can exert on the ground, in N/m^2.

1 m^2 = 10 000 cm^2, so 300 cm^2 = 0.03 m^2 and 600 cm^2 = 0.06 m^2
(a) Greatest pressure is on the smallest face: 420 ÷ 0.03 = 14,000 N/m^2
(b) Least pressure is on the largest face: 420 ÷ 0.06 = 7,000 N/m^2
Answer: (a) 14,000 N/m^2 (b) 7,000 N/m^2

3 Circle theorems: angle at the centre

A, B and C are points on a circle, centre O. Angle OAB = 45°.
(a) Work out the size of angle AOB.
(b) Work out the size of angle ACB.
The diagram is not drawn accurately.

(a) OA = OB (radii), so triangle OAB is isosceles and angle OBA = 45°
Angle AOB = 180 − 2 × 45 = 90°
(b) The angle at the circumference is half the angle at the centre: 90 ÷ 2 = 45°
Answer: (a) 90° (b) 45°

4 Direct proportion formulae

y is directly proportional to the square of x.
When x = 3, y = 45.
(a) Find a formula for y in terms of x.
(b) Work out the value of y when x = 5.

(a) $y = kx^2$
45 = k × 9, so k = 45 ÷ 9 = 5
$y = 5x^2$
(b) y = 5 × 25 = 125
Answer: (a) $y = 5x^2$ (b) 125

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5 Perimeter and area of a running track

The diagram shows a running track made from a rectangle and two semicircles.
(a) Work out the total length of the boundary of the track.
(b) Work out the area enclosed by the track. Give your answers in terms of π.
The diagram is not drawn accurately.

(a) The two semicircles make one full circle of radius 15: circumference = $2 \times \pi \times 15 = 30\pi$
The two straight sides: $2 \times 100 = 200$
Boundary = $(200 + 30\pi)$ m
(b) The rectangle is 100 m by $2 \times 15 = 30$ m, so its area is $100 \times 30 = 3000$. Circle: $\pi \times 15^2 = 225\pi$
Area = $(3000 + 225\pi)$ m^2

Answer: (a) $(200 + 30\pi)$ m (b) $(3000 + 225\pi)$ m^2

6 Fractions of fractions

Uma spent $\frac{5}{8}$ of her money on a coat.

She then spent $\frac{1}{6}$ of the money that was left on a bag.

She then had £150 left.
How much money did Uma have to start with?

After the first part, $\frac{3}{8}$ remains

After the second, $\frac{5}{6}$ of $\frac{3}{8}$ remains = $\frac{5}{16}$ of the total

$\frac{5}{16}$ of the total is £150

Total = £150 ÷ $\frac{5}{16}$ = £480

Answer: £480

7 Pythagoras and quadratic equations

The lengths of the sides of a right-angled triangle are $(x - 4)$ cm, $(x + 3)$ cm and $(x + 4)$ cm.
The longest side is $(x + 4)$ cm.
(a) Show that $x^2 - 10x + 9 = 0$
(b) Hence work out the area of the triangle.

(a) Pythagoras: $(x - 4)^2 + (x + 3)^2 = (x + 4)^2$
Expand: $x^2 - 8x + 16 + x^2 + 6x + 9 = x^2 + 8x + 16$
Rearrange: $x^2 - 10x + 9 = 0$
(b) Factorise: $(x - 9)(x - 1) = 0$, so $x = 9$ (the other root, $x = 1$, does not give valid side lengths)

The shorter sides are 5 cm and 12 cm, so area = $\frac{1}{2} \times 5 \times 12 = 30$ cm^2

Answer: (b) 30 cm^2

8 Composite functions: fg and gf

$f(x) = -3x - 1$ and $g(x) = x^2 - 2$
Solve $fg(x) = gf(x)$

$fg(x) = f(x^2 - 2) = -3x^2 + 5$
$gf(x) = g(-3x - 1) = (-3x - 1)^2 - 2 = 9x^2 + 6x - 1$
Set them equal and rearrange: $12x^2 + 6x - 6 = 0$
Divide by 6 and factorise: $(x + 1)(2x - 1) = 0$

$x = -1$ or $x = \frac{1}{2}$

Answer: $x = -1$ or $x = \frac{1}{2}$

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9 Frustum of a cone

A cone is cut parallel to its base to make a smaller cone and a frustum.
The frustum has height 6 cm, top radius 2 cm and base radius 4 cm.
Work out the volume of the frustum. Give your answer in terms of π.

Let the height of the small cone removed be h cm. The cones are similar, so $\frac{h}{h+6} = \frac{2}{4}$

$4h = 2(h + 6)$, so $2h = 12$ and $h = 6$

Large cone: $\frac{1}{3} \times \pi \times 4^2 \times 12 = 64\pi$

Small cone: $\frac{1}{3} \times \pi \times 2^2 \times 6 = 8\pi$

Frustum = $64\pi - 8\pi = 56\pi$ cm^3

Answer: 56π cm^3

10 Surds and fractions

Simplify $\frac{1}{7+\sqrt{7}} + \frac{1}{7-\sqrt{7}}$

Give your answer with a rational denominator.

Common denominator: $(7 + \sqrt{7})(7 - \sqrt{7}) = 49 - 7 = 42$

Numerator: $(7 - \sqrt{7}) + (7 + \sqrt{7})$

$= 14$

$\frac{14}{42} = \frac{1}{3}$

Answer: $\frac{1}{3}$