Day 71 **S Tier 10-a-day: Model Solutions**

1 3D Pythagoras and trigonometry: cuboids

The diagram shows a cuboid ABCDEFGH. (Calculator)
(a) Work out the length of the diagonal AG.
(b) Work out the angle between AG and the base ABCD, to 1 decimal place.
The diagram is not drawn accurately.

(a) $AC^2 = AB^2 + BC^2 = 14^2 + 8^2 = 260$
$AG^2 = AC^2 + CG^2 = 260 + 8^2 = 324$
AG = 18 cm

(b) The angle is GAC. $\tan(GAC) = \frac{CG}{AC} = \frac{8}{\sqrt{260}}$

angle GAC = 26.4° (1 d.p.)
Answer: (a) 18 cm (b) 26.4°

2 Factorising fully

Factorise fully $25x^2 - 49y^2$

$25x^2 - 49y^2$ is a difference of two squares: $(5x)^2 - (7y)^2$
$= (5x - 7y)(5x + 7y)$
Answer: (5x − 7y)(5x + 7y)

3 Tree diagrams without replacement

A bag contains 6 orange cubes, 2 purple cubes and 3 pink cubes.
Two cubes are taken at random without replacement.
Work out the probability that (a) at least one is purple, (b) exactly one is purple.

(a) P(at least one purple) = 1 − P(none) = $1 - \frac{9}{11} \times \frac{8}{10} = \frac{19}{55}$

(b) P(exactly one purple) = $\frac{2}{11} \times \frac{9}{10} + \frac{9}{11} \times \frac{2}{10} = \frac{18}{55}$

Answer: (a) $\frac{19}{55}$ (b) $\frac{18}{55}$

4 Laws of indices with algebra

Simplify $(27x^3y^6)^{\frac{4}{3}}$

Raise each part to the power $\frac{4}{3}$

$27^{\frac{4}{3}} = (\sqrt[3]{27})^4 = 3^4 = 81$
$(x^3)^{\frac{4}{3}} = x^4$ and $(y^6)^{\frac{4}{3}} = y^8$
$= 81x^4y^8$
Answer: $81x^4y^8$

5 Area and quadratic equations

The diagram shows a rectangle of length (x − 2) cm, width (x + 1) cm and area 54 cm^2.
(a) Show that $x^2 - x - 56 = 0$.
(b) Work out the perimeter of the rectangle.

(a) $(x - 2)(x + 1) = 54$
Expand: $x^2 - x - 2 = 54$, so $x^2 - x - 56 = 0$
(b) Factorise: $(x - 8)(x + 7) = 0$
x = 8 (x = −7 is not possible as a length would be negative)
Length = 6 cm and width = 9 cm
Perimeter = 2 × (6 + 9) = 30 cm
Answer: (b) 30 cm

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6 Using a line of best fit

The scatter graph shows distance from town (x) and price (y).
The equation of the line of best fit is $y = -1.5x + 36$.
(a) Estimate y when x = 3.
(b) Explain why an estimate of y when x = 15 may be unreliable.

(a) $y = -1.5 \times 3 + 36 = 31.5$
(b) x = 15 is outside the range of the data on the graph, so the estimate relies on the pattern continuing (extrapolation)
Answer: (a) 31.5 (b) x = 15 is outside the range of the data

7 Rationalising the denominator

Write $\frac{8}{2+\sqrt{10}}$ in the form $\frac{p+q\sqrt{10}}{r}$
where p, q and r are integers and r > 1,
and the fraction is in its simplest form.

Multiply top and bottom by $(2 - \sqrt{10})$:
$\frac{8}{2+\sqrt{10}} = \frac{8(2-\sqrt{10})}{(2+\sqrt{10})(2-\sqrt{10})}$
Denominator: $4 - 10 = -6$
$= \frac{-8+4\sqrt{10}}{3}$

Answer: $\mathbf{\frac{-8+4\sqrt{10}}{3}}$

8 Equal roots and real roots

The equation $x^2 + kx + 5 = 0$ has exactly one solution.
Find the possible values of k.
Give any surds in their simplest form.

One solution means $b^2 - 4ac = 0$
$k^2 - 4 \times 1 \times 5 = 0$, so $k^2 = 20$
$k = \pm\sqrt{20} = \pm 2\sqrt{5}$
Answer: k = $\pm 2\sqrt{5}$

9 Combined transformations

The point P(−3, 1) is rotated 180° about the origin.
The image is then reflected in the y-axis.
(a) Write down the coordinates of the final image of P.
(b) Describe fully the single transformation that is equivalent to these two transformations.

(a) After the first transformation: P' = (3, −1)
After the second transformation: P'' = (−3, −1)
(b) Follow a general point (x, y) through both transformations: (x, y) → (−x, −y) → (x, −y)
The single mapping is (x, y) → (x, −y)
This is a reflection in the x-axis
Answer: (a) (−3, −1) (b) Reflection in the x-axis

10 Squares, cubes and prime factors

Find the smallest positive integer k such that 35 × k is a cube number.

35 = 5 × 7
For a cube number every power must be a multiple of 3
Multiply by $5^2 \times 7^2$ to get $5^3 \times 7^3$
k = 1225
Answer: k = 1225