Day 70 **S Tier 10-a-day: Model Solutions**

1 Ordering numbers

Write these numbers in order of size, starting with the smallest.

$\frac{1}{4}$, $\frac{8}{9}$, $\frac{3}{8}$, 0.78

Convert each to a decimal: $\frac{1}{4} = 0.25$, $\frac{8}{9} = 0.889$, $\frac{3}{8} = 0.375$, $0.78 = 0.78$

In order: $\frac{1}{4}$, $\frac{3}{8}$, 0.78, $\frac{8}{9}$

Answer: $\frac{1}{4}$, $\frac{3}{8}$, 0.78, $\frac{8}{9}$

2 Speed-time graphs

The graph shows the speed of a train between two stations.
(a) Work out the acceleration in the first 4 seconds.
(b) Work out the total distance.

(a) Acceleration = change in speed ÷ time = 16 ÷ 4 = 4 m/s^2
(b) Distance = area under the graph, split into two triangles and a rectangle
$\frac{1}{2} \times 4 \times 16 + 16 \times 15 + \frac{1}{2} \times 4 \times 16 = 32 + 240 + 32$
= 304

Answer: (a) 4 m/s^2 (b) 304 m

3 Radius from chord and height of arc

A circular arc has a chord AB of length 36 cm.
The greatest distance between the chord and the arc is 6 cm.
Work out the radius of the circle.

Let the radius be r. The centre O is on the line through the midpoint M of AB and the highest point of the arc, so OM = r − 6
AM = 36 ÷ 2 = 18. Pythagoras in triangle OMA: $r^2 = (r - 6)^2 + 18^2$
$r^2 = r^2 - 12r + 36 + 324$
12r = 360, so r = 30 cm

Answer: 30 cm

4 Fraction problems

A bath is $\frac{3}{4}$ full.

10 more litres of water are added and the bath is then $\frac{4}{5}$ full.

Work out the capacity of the bath.

Fraction added = $\frac{4}{5} - \frac{3}{4} = \frac{1}{20}$

$\frac{1}{20}$ of the capacity is 10 litres

Capacity = $10 \div \frac{1}{20} = 200$

Answer: 200 litres

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5 Rates of work

A tank is filled by a tap in 15 minutes.
A drain can empty the full tank in 30 minutes.
The tap and the drain are both open, and the tank is empty to begin with.
How long does it take for the tank to fill?

In 1 minute: the tap fills $\frac{1}{15}$ and the drain empties $\frac{1}{30}$

Net: $\frac{1}{15} - \frac{1}{30} = \frac{15}{450}$

Time = $\frac{450}{15}$ = 30 minutes

Answer: 30 minutes

6 Alternate segment theorem

PAQ is a tangent to the circle at A. B and C are points on the circle. Angle PAB = (3x + 10)°, angle ACB = (2x + 25)° and angle CAQ = 38°.
(a) Work out the value of x.
(b) Work out the size of angle BAC.
The diagram is not drawn accurately.

(a) Angle ACB = angle PAB (alternate segment theorem)
2x + 25 = 3x + 10
x = 15
(b) Angle PAB = 55°
Angles on the straight line PAQ: angle BAC = 180 − 55 − 38 = 87°
Answer: (a) x = 15 (b) 87°

7 Mixtures and percentage profit

A shop mixes two types of tea.
Type A costs £6.00 per kg and type B costs £14.00 per kg.
They are mixed in the ratio A : B = 3 : 5.
The mixture is sold for £13.75 per kg.
Work out the percentage profit.

Cost of 8 kg of mixture = 3 × £6.00 + 5 × £14.00 = £88.00
Cost per kg = £88.00 ÷ 8 = £11.00
Profit per kg = £13.75 − £11.00 = £2.75
Percentage profit = 2.75 ÷ 11.00 × 100 = 25%
Answer: 25%

8 Tangent to a circle

A circle has equation $x^2 + y^2 = 130$.
The point P(−9, 7) lies on the circle.
Find an equation of the tangent to the circle at P.
Give your answer in the form ax + by = c, where a, b and c are integers.

The centre is (0, 0). The gradient of the radius to P is $\frac{7-0}{-9-0} = -\frac{7}{9}$

The tangent is perpendicular to the radius, so its gradient is $\frac{9}{7}$. Using P: $y - 7 = \frac{9}{7}(x + 9)$

Multiply by 7: $7(y - 7) = 9(x + 9)$
Expand and rearrange: $-9x + 7y = 130$
Multiply by −1: $9x - 7y = -130$
Answer: $9x - 7y = -130$

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9 Counting factors

$N = 2^4 \times 3^2 \times 7^2$

How many factors does N have?

Include 1 and N.

Add 1 to each power and multiply

$(4 + 1) \times (2 + 1) \times (2 + 1) = 5 \times 3 \times 3 = 45$

Answer: 45

10 Probability and quadratic equations

A bag contains some sweets. 3 of them are red and the rest are yellow.

Two sweets are taken at random without replacement.

The probability that both are red is $\frac{1}{7}$.

Work out how many sweets are yellow.

Let there be n sweets altogether

$P(\text{both red}) = \frac{3}{n} \times \frac{2}{n-1} = \frac{6}{n(n-1)}$

$\frac{6}{n(n-1)} = \frac{1}{7}$, so $n(n-1) = 42$, giving $n^2 - n - 42 = 0$

$(n - 7)(n + 6) = 0$, so $n = 7$ (n cannot be negative)

Number of sweets that are yellow $= 7 - 3 = 4$

Answer: 4