Day 7 **S Tier 10-a-day: Model Solutions**

1 Angles in polygons

Each interior angle of a regular polygon is 4 times the size of each exterior angle.
Work out the number of sides of the polygon.

Let each exterior angle be e. Then each interior angle is 4e.
Interior angle + exterior angle = 180°, so 4e + e = 180°
e = 180° ÷ 5 = 36°
Number of sides = 360° ÷ 36° = 10
Answer: 10

2 Weighted percentages

The ratio of Year 10 students to Year 11 students in a school is 3 : 5.
25% of the Year 10 students and 65% of the Year 11 students have a part-time job.
What percentage of all of these students have a part-time job?

Imagine 160 students in total: 60 Year 10 students and 100 Year 11 students
25% of 60 = 15 and 65% of 100 = 65
Together: 80 out of 160
80 ÷ 160 × 100 = 50%
Answer: 50%

3 Multiplying and dividing surds

Simplify fully $2\sqrt{7} \times \sqrt{3}$

$2\sqrt{7} \times \sqrt{3} = 2\sqrt{21}$
Answer: $2\sqrt{21}$

4 Inverse proportion

P is inversely proportional to the square root of t.
When t = 16, P = 2.
Find the positive value of t when P = 8.

$P = \frac{k}{\sqrt{t}}$
$2 = \frac{k}{4}$, so $k = 2 \times 4 = 8$
$8 = \frac{8}{\sqrt{t}}$, so $\sqrt{t} = \frac{8}{8} = 1$
$t = 1^2 = 1$
Answer: t = 1

5 Percentages

C is 40% less than D.
D is 25% more than S.
Work out C as a percentage of S.

C = 0.6 × D and D = 1.25 × S
C = 0.6 × 1.25 × S = 0.75 × S
0.75 × 100 = 75%
Answer: 75%

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6 Bounds in geometry

The radius of a circle is 4 cm, correct to the nearest centimetre.
Work out the upper bound of the area of the circle. Give your answer in terms of π.

The radius is between $\frac{7}{2}$ cm and $\frac{9}{2}$ cm

For the upper bound, use r = $\frac{9}{2}$

Area = $\pi \times (\frac{9}{2})^2 = \frac{81}{4}\pi$ cm^2

Answer: $\frac{81}{4}\pi$ cm^2

7 Pythagoras and quadratic equations

The lengths of the sides of a right-angled triangle are (x − 1) cm, (x + 6) cm and (x + 8) cm.
The longest side is (x + 8) cm.
(a) Show that $x^2 - 6x - 27 = 0$
(b) Hence work out the area of the triangle.

(a) Pythagoras: $(x - 1)^2 + (x + 6)^2 = (x + 8)^2$
Expand: $x^2 - 2x + 1 + x^2 + 12x + 36 = x^2 + 16x + 64$
Rearrange: $x^2 - 6x - 27 = 0$
(b) Factorise: $(x - 9)(x + 3) = 0$, so x = 9 (the other root, x = −3, does not give valid side lengths)
The shorter sides are 8 cm and 15 cm, so area = $\frac{1}{2} \times 8 \times 15 = 60$ cm^2

Answer: (b) 60 cm^2

8 Equation of a curve from a sketch

The diagram shows a sketch of a quadratic curve with a turning point at (4, −4).
The curve crosses the y-axis at (0, 28).
Find an equation of the curve.
Give your answer in the form $y = ax^2 + bx + c$.

The turning point is (4, −4), so $y = a(x - 4)^2 - 4$
When x = 0, y = 28: a × 16 − 4 = 28, so a = 2
$y = 2(x - 4)^2 - 4$
Expand: $y = 2x^2 - 16x + 28$
Answer: $y = 2x^2 - 16x + 28$

9 Bounds of speed, density and pressure

A force of 87 N acts on a surface, correct to the nearest whole number.
The area of the surface is 24 m^2, correct to the nearest whole number.
Work out the upper bound of the pressure.
Give your answer correct to 3 significant figures. (Calculator)

86.5 ≤ force < 87.5; 23.5 ≤ area < 24.5
Upper bound of pressure = biggest force ÷ smallest area = 87.5 ÷ 23.5
= 3.72340... = 3.72 (3 s.f.)
Answer: 3.72 N/m^2

10 Terms of a quadratic sequence

The nth term of a sequence is $3n^2 - 2n - 8$.
(a) Work out the 8th term.
(b) Is 125 a term of the sequence? You must show your working.

(a) $3 \times 8^2 - 2 \times 8 - 8 = 168$

(b) Solve $3n^2 - 2n - 8 = 125$, so $3n^2 - 2n - 133 = 0$

Factorise or use the formula: $n = 7$ or $n = -\frac{19}{3}$

n = 7 is a positive whole number, so 125 is term number 7

Answer: (a) 168 (b) Yes, it is the 7th term