Day 69 **S Tier 10-a-day: Model Solutions**

1 Arc length and sector area

A sector of a circle has angle 120° and area 27π cm^2.
(a) Work out the radius of the circle.
(b) Work out the length of the arc of the sector. Give your answer to (b) in terms of π.

(a) Area = $\frac{120}{360} \times \pi \times r^2$, so $\frac{120}{360} \times r^2 = 27$

$r^2 = 27 \times \frac{360}{120} = 81$

$r = 9$ cm

(b) Arc length = $\frac{120}{360} \times 2 \times \pi \times 9 = 6\pi$ cm

Answer: (a) 9 cm (b) 6π cm

2 LCM problems

A red light flashes every 6 minutes.
A blue light flashes every 15 minutes.
A green light flashes every 18 minutes.
They all flash together at 8:00 am.
How many times do they all flash together between 8:00 am and 3:00 pm, including both 8:00 am and 3:00 pm if they do?

Find the LCM of 6, 15 and 18

$6 = 2 \times 3$, $15 = 3 \times 5$, $18 = 2 \times 3^2$

LCM = $2 \times 3^2 \times 5 = 90$

Total time from 8:00 am to 3:00 pm = 420 minutes

$420 \div 90 = 4.67$, so there are 4 more times after the start

Including the start: $4 + 1 = 5$

Answer: 5

3 Shaded areas: exact answers

The diagram shows a square inside a circle of radius 6 cm.
The four vertices of the square lie on the circle.
Work out the area of the shaded region. Give your answer in terms of π.

The diagonal of the square is a diameter of the circle: 12 cm

Area of the square = $\frac{1}{2} \times$ diagonal $\times$ diagonal = $\frac{1}{2} \times 12 \times 12 = 72$

Area of circle = $\pi \times 6^2 = 36\pi$

Shaded area = $36\pi - 72$ cm^2

Answer: $(36\pi - 72)$ cm^2

4 Direct proportion formulae

y is directly proportional to the cube of x.
When x = 1, y = 5.
(a) Find a formula for y in terms of x.
(b) Work out the positive value of x when y = 40.

(a) $y = kx^3$

$5 = k \times 1$, so $k = 5 \div 1 = 5$

$y = 5x^3$

(b) $40 = 5x^3$, so $x^3 = 40 \div 5 = 8$, giving $x = 2$

Answer: (a) $y = 5x^3$ (b) $x = 2$

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5 Alternate segment theorem

PAQ is a tangent to the circle at A. B and C are points on the circle. Angle PAB = (2x + 15)°, angle ACB = (3x + 4)° and angle CAQ = 44°.
(a) Work out the value of x.
(b) Work out the size of angle BAC.
The diagram is not drawn accurately.

(a) Angle ACB = angle PAB (alternate segment theorem)
3x + 4 = 2x + 15
x = 11
(b) Angle PAB = 37°
Angles on the straight line PAQ: angle BAC = 180 − 37 − 44 = 99°
Answer: (a) x = 11 (b) 99°

6 Rationalising the denominator

Write $\frac{14}{2+\sqrt{5}}$ in the form $p + q\sqrt{5}$
where p and q are integers.

Multiply top and bottom by $(2 - \sqrt{5})$:
$\frac{14}{2+\sqrt{5}} = \frac{14(2-\sqrt{5})}{(2+\sqrt{5})(2-\sqrt{5})}$
Denominator: $4 - 5 = -1$
$= -28 + 14\sqrt{5}$
Answer: $-28 + 14\sqrt{5}$

7 Algebraic fractions: multiplying and dividing

Simplify fully $\frac{x^2-9}{x^2-6x} \div \frac{x^2-7x+12}{x}$

Factorise: $\frac{(x-3)(x+3)}{x(x-6)} \div \frac{(x-4)(x-3)}{x}$

Turn the second fraction upside down and multiply: $\frac{(x-3)(x+3)}{x(x-6)} \times \frac{x}{(x-4)(x-3)}$

Cancel $(x - 3)$ and x
$= \frac{(x+3)}{(x-6)(x-4)}$
Answer: $\frac{(x+3)}{(x-6)(x-4)}$

8 Line of best fit

The scatter graph shows distance from town (x) and price (y).
The line of best fit passes through (2, 18) and (6, 16).
(a) Find its equation in the form y = mx + c.
(b) Explain what its gradient tells you.

(a) Gradient $m = \frac{16-18}{6-2} = \frac{-2}{4} = -0.5$
$y = -0.5x + c$: using (2, 18), $18 = -0.5 \times 2 + c$, so $c = 19$
$y = -0.5x + 19$
(b) For each extra kilometre from the town, the house price (in £10 000s) goes down by 0.5
Answer: (a) $y = -0.5x + 19$ (b) For each extra kilometre from the town, the house price (in £10 000s) goes down by 0.5

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9 Tangent to a circle: proof

Show that the line $y = 3x + 10$ is a tangent to the circle $x^2 + y^2 = 10$.
Find the coordinates of the point where the line touches the circle.

Substitute $y = 3x + 10$ into $x^2 + y^2 = 10$: $x^2 + (3x + 10)^2 = 10$

Expand: $10x^2 + 60x + 90 = 0$

$b^2 - 4ac = 60^2 - 4 \times 10 \times 90 = 3600 - 3600 = 0$

$b^2 - 4ac = 0$, so there is exactly one point of intersection: the line is a tangent

$x = \frac{-60}{2 \times 10} = -3$, and $y = 3 \times (-3) + 10 = 1$. The point is $(-3, 1)$

Answer: $(-3, 1)$

10 Joint and inverse variation

F is directly proportional to m and inversely proportional to the square of d.
When m = 5 and d = 1, F = 5.
Work out the value of F when m = 3 and d = 1.

$F = \frac{km}{d^2}$

$5 = \frac{k \times 5}{1}$, so $k = 5 \times 1 \div 5 = 1$

$F = \frac{1 \times 3}{1} = 3$

Answer: F = 3