Day 68 **S Tier 10-a-day: Model Solutions**

1 Substitution

Work out the value of $abc - a^2$
when $a = 4$, $b = -2$ and $c = -1$.

Substitute the values: $4 \times (-2) \times (-1) - 4^2$
$= 8 - 16$
$= -8$
Answer: -8

2 Fraction problems

A tank is $\frac{2}{5}$ full.

16 more litres of water are added and the tank is then $\frac{2}{3}$ full.

Work out the capacity of the tank.

Fraction added = $\frac{2}{3} - \frac{2}{5} = \frac{4}{15}$

$\frac{4}{15}$ of the capacity is 16 litres

Capacity = $16 \div \frac{4}{15} = 60$

Answer: 60 litres

3 3D Pythagoras and trigonometry: cuboids

The diagram shows a cuboid ABCDEFGH. (Calculator)
(a) Work out the length of the diagonal AG.
(b) Work out the angle between AG and the base ABCD, to 1 decimal place.
The diagram is not drawn accurately.

(a) $AC^2 = AB^2 + BC^2 = 3^2 + 12^2 = 153$
$AG^2 = AC^2 + CG^2 = 153 + 4^2 = 169$
$AG = 13$ cm

(b) The angle is GAC. $\tan(GAC) = \frac{CG}{AC} = \frac{4}{\sqrt{153}}$

angle GAC = 17.9° (1 d.p.)
Answer: (a) 13 cm (b) 17.9°

4 Mean of a changing set

The mean of 5 numbers is 20.
One number is removed and the mean of the remaining 4 numbers is 17.5.
Work out the number that was removed.

Total of the 5 numbers = $5 \times 20 = 100$
Total of the 4 remaining = $4 \times 17.5 = 70$
Number removed = $100 - 70 = 30$
Answer: 30

5 Percentages

P is 5% less than A.
A is 80% less than R.
Work out P as a percentage of R.

$P = 0.95 \times A$ and $A = 0.2 \times R$
$P = 0.95 \times 0.2 \times R = 0.19 \times R$
$0.19 \times 100 = 19\%$
Answer: 19%

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6 Angles of elevation

A point A is on level ground in line with the base of a tower. (Calculator)
The angle of elevation of the top of the tower from A is 45°.
B is a point on the same line, 25 m further from the tower than A.
The angle of elevation of the top of the tower from B is 25°.
Work out the height of the tower. Give your answer to 1 decimal place.

Let the height be h and the distance from A to the base be x. $\tan 45° = \frac{h}{x}$, so $x = h \div \tan 45°$

From B: $\tan 25° = \frac{h}{x + 25}$, so $x + 25 = h \div \tan 25°$

Subtract: $25 = h \div \tan 25° - h \div \tan 45° = h(2.1445 - 1.0000)$
$h = 25 \div 1.1445 = 21.8$ m (1 d.p.)
Answer: 21.8 m

7 Equations with algebraic fractions

Solve $\frac{4}{x - 5} - \frac{2}{x - 2} = 1$

Multiply every term by $(x - 5)(x - 2)$: $4(x - 2) - 2(x - 5) = (x - 5)(x - 2)$
Expand and rearrange so that one side is 0: $x^2 - 9x + 8 = 0$
Factorise: $(x - 8)(x - 1) = 0$
$x = 1$ or $x = 8$
Answer: x = 1 or x = 8

8 Perpendicular bisector

A is the point (−2, 4) and B is the point (−6, 6).
Find an equation of the perpendicular bisector of AB.
Give your answer in the form y = mx + c.

Midpoint of AB = $(\frac{-2 + (-6)}{2}, \frac{4 + 6}{2}) = (-4, 5)$

Gradient of AB = $\frac{6 - 4}{-6 - (-2)} = -\frac{1}{2}$

The perpendicular gradient is $-1 \div (-\frac{1}{2}) = 2$

Using the midpoint: $y - 5 = 2(x + 4)$
$y = 2x + 13$
Answer: y = 2x + 13

9 Loci and regions

ABCD is a rectangle with AB = 8 cm and AD = 10 cm.
A point P inside the rectangle is such that AP ≤ 8 cm and P is nearer to A than to B.
Work out the area of the region in which P can lie. Give your answer in terms of π and surds.

Nearer to A than to B: P is on the A side of the perpendicular bisector of AB, a distance 4 cm from A
AP ≤ 8 cm: P is inside the quarter circle of radius 8 cm centred at A

The bisector meets the arc where $\cos \theta = \frac{4}{8}$, so $\theta = 60°$ at A. The region is a right-angled triangle plus a sector of angle 30°

Triangle: base 4 cm, height $8 \sin 60° = 4\sqrt{3}$ cm, area $= \frac{1}{2} \times 4 \times 4\sqrt{3} = 8\sqrt{3}$

Sector of angle 30° : $\frac{30}{360} \times \pi \times 8^2 = \frac{16}{3}\pi$

Area $= 8\sqrt{3} + \frac{16}{3}\pi$ cm^2

Answer: $(8\sqrt{3} + \frac{16}{3}\pi)$ cm^2

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10 Combining means

The mean of the marks in a test for the men is 76.
The mean of the marks in a test for the women is 59.
The mean for all of the men and women together is 70.9.
Work out the ratio of the number of men to the number of women.

Let there be p men and q women
Total = 76p + 59q, and the overall mean gives 76p + 59q = 70.9(p + q)
5.1p = 11.9q
p : q = 11.9 : 5.1 = 7 : 3
Answer: 7 : 3