Day 67 **S Tier 10-a-day: Model Solutions**

1 Fraction problems

A bath is $\frac{7}{10}$ full.

18 more litres of water are added and the bath is then $\frac{3}{4}$ full.

Work out the capacity of the bath.

Fraction added = $\frac{3}{4} - \frac{7}{10} = \frac{1}{20}$

$\frac{1}{20}$ of the capacity is 18 litres

Capacity = $18 \div \frac{1}{20} = 360$

Answer: 360 litres

2 Tangents and circle theorems

PA and PB are tangents to a circle, centre O. C is a point on the circle. Angle ACB = 49°.
(a) Work out the size of angle AOB.
(b) Work out the size of angle APB.
The diagram is not drawn accurately.

(a) The angle at the centre is twice the angle at the circumference: $2 \times 49 = 98°$
(b) Angles OAP and OBP are 90° (a tangent meets a radius at 90°)
Angle APB = $360 - 90 - 90 - 98 = 82°$
Answer: (a) 98° (b) 82°

3 Factorising fully

Factorise fully $(x + 4)^2 - 25$

This is a difference of two squares: $(x + 4)^2 - 5^2$
$= (x + 4 - 5)(x + 4 + 5)$
$= (x - 1)(x + 9)$
Answer: $(x - 1)(x + 9)$

4 Alternate segment theorem

PAQ is a tangent to the circle at A. B and C are points on the circle. Angle PAB = 47° and angle CAQ = 74°.
(a) Work out the size of angle ABC.
(b) Work out the size of angle ACB.
The diagram is not drawn accurately.

(a) Angle ABC = angle CAQ = 74° (alternate segment theorem)
(b) Angle ACB = angle PAB = 47° (alternate segment theorem)
Answer: (a) 74° (b) 47°

5 Three-part ratios

Isla, Jamal and Kira share some money.
Isla : Jamal = 8 : 5
Jamal : Kira = 3 : 5
(a) Work out the ratio Isla : Jamal : Kira.
(b) They share £128 altogether. How much money does Isla get?

(a) Make the Jamal part the same: 8 : 5 = 24 : 15 and 3 : 5 = 15 : 25
Isla : Jamal : Kira = 24 : 15 : 25
(b) Total parts = 64, so 1 part = $128 \div 64 = 2$
Isla gets $24 \times 2 = £48$
Answer: (a) 24 : 15 : 25 (b) £48

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6 Percentages

C is 30% less than S.
S is 30% more than Q.
Work out C as a percentage of Q.

C = 0.7 × S and S = 1.3 × Q
C = 0.7 × 1.3 × Q = 0.91 × Q
0.91 × 100 = 91%
Answer: 91%

7 Exponential growth and decay

There are 20 bacteria in a sample. The number of bacteria is multiplied by 4 every hour.
After how many hours will there first be more than 10,000 bacteria?

Number after n hours = 20×4^n
n = 4: 5,120
n = 5: 20,480
20,480 is the first value above 10,000
Answer: 5 hours

8 Percentage change and proportion

The time taken for a journey is inversely proportional to the speed.
The speed is increased by 100%.
Work out the percentage change in the time taken for a journey.

The speed is multiplied by 2
The multiplier for the time taken for a journey is 1 ÷ 2 = 0.5
0.5 is a decrease of 50%
Answer: 50% decrease

9 Angles of elevation

A point A is on level ground in line with the base of a mast. (Calculator)
The angle of elevation of the top of the mast from A is 38°.
B is a point on the same line, 80 m further from the mast than A.
The angle of elevation of the top of the mast from B is 21°.
Work out the height of the mast. Give your answer to 1 decimal place.

Let the height be h and the distance from A to the base be x. $\tan 38° = \frac{h}{x}$, so $x = h \div \tan 38°$

From B: $\tan 21° = \frac{h}{x + 80}$, so $x + 80 = h \div \tan 21°$

Subtract: $80 = h \div \tan 21° - h \div \tan 38° = h(2.6051 - 1.2799)$
h = 80 ÷ 1.3251 = 60.4 m (1 d.p.)
Answer: 60.4 m

10 Tangency conditions

The line $y = kx - 5$ is a tangent to the curve $y = x^2 + 3$.
Find the possible values of k.
Give any surds in their simplest form.

Equate: $x^2 + 3 = kx - 5$
Rearrange: $x^2 - kx + 8 = 0$
One point of contact, so $b^2 - 4ac = 0$: $k^2 - 4 \times 8 = 0$
$k^2 = 32$
$k = \pm\sqrt{32} = \pm 4\sqrt{2}$
Answer: $k = \pm 4\sqrt{2}$