Day 66 **S Tier 10-a-day: Model Solutions**

1 Completing the square

(a) Write $4x^2 + 40x + 105$ in the form $a(x + p)^2 + q$

(b) Write down the coordinates of the turning point of the curve $y = 4x^2 + 40x + 105$

(a) Take out the factor 4 from the x terms: $4(x^2 + 10x) + 105$

$x^2 + 10x = (x + 5)^2 - 25$, so $4x^2 + 40x + 105 = 4[(x + 5)^2 - 25] + 105$

$= 4(x + 5)^2 + 5$

(b) The minimum is when $x + 5 = 0$, so $x = -5$ and $y = 5$

Answer: (a) $4(x + 5)^2 + 5$ (b) $(-5, 5)$

2 Mean of a changing set

The mean of 5 numbers is 47.

One number is removed and the mean of the remaining 4 numbers is 51.75.

Work out the number that was removed.

Total of the 5 numbers = $5 \times 47 = 235$

Total of the 4 remaining = $4 \times 51.75 = 207$

Number removed = $235 - 207 = 28$

Answer: 28

3 HCF and LCM

(a) Write 189 and 72 as products of prime factors.

(b) Find the HCF and the LCM of 189 and 72.

Give your answers as numbers.

(a) $189 = 3^3 \times 7$ and $72 = 2^3 \times 3^2$

(b) HCF = $3^2 = 9$

LCM = $2^3 \times 3^3 \times 7 = 1512$

Answer: (a) $3^3 \times 7$; $2^3 \times 3^2$ (b) HCF = 9, LCM = 1512

4 Capture and recapture

40 beetles in a field are caught, marked and put back.

Later, a second sample of 40 beetles is caught. 2 of these are marked.

(a) Work out an estimate for the number of beetles in a field.

(b) State one assumption you have made.

(a) The proportion marked in the second sample is 2 out of 40

This should match the proportion marked in the whole population: $\frac{2}{40} = \frac{40}{N}$

$N = 40 \times 40 \div 2 = 800$

(b) No beetles are born, die or leave between the samples, and the beetles mix evenly

Answer: (a) 800 (b) The population stays the same between samples

5 Angle in a semicircle

AB is a diameter of a circle, centre O. C is a point on the circle. Angle BAC = 24°.

(a) Work out the size of angle ACB.

(b) Work out the size of angle COB.

The diagram is not drawn accurately.

(a) The angle in a semicircle is 90°, so angle ACB = 90°

(b) OA = OC (radii), so triangle OAC is isosceles and angle OCA = 24°

Angle COB is an exterior angle of triangle OAC: 24 + 24 = 48°

(or: the angle at the centre is twice the angle at the circumference)

Answer: (a) 90° (b) 48°

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6 Iteration

Use the iteration formula $x_{n+1} = \sqrt[3]{6x_n + 6}$ with $x_0 = 1$ (Calculator)
to work out x_1, x_2 and x_3.
Give each answer correct to 3 decimal places.

$x_1 = \sqrt[3]{6 \times 1 + 6} = 2.28943...$

$x_2 = \sqrt[3]{6 \times x_1 + 6} = 2.70245...$

$x_3 = \sqrt[3]{6 \times x_2 + 6} = 2.81112...$

Rounded: $x_1 = 2.289$, $x_2 = 2.702$, $x_3 = 2.811$

Answer: $x_1 = 2.289$, $x_2 = 2.702$, $x_3 = 2.811$

7 Exact lengths and areas

A square has sides of length $(5 + \sqrt{3})$ cm.
Work out the area of the square.
Give your answer in the form $a + b\sqrt{3}$

Area $= (5 + \sqrt{3})^2$

$= 25 + 10\sqrt{3} + 3$

$= 28 + 10\sqrt{3}$

Answer: $28 + 10\sqrt{3}$ cm^2

8 Algebraic fractions: adding and subtracting

Show that $\frac{x+1}{x-5} - \frac{5}{x-3}$ can be written as $\frac{x^2 - 7x + 22}{(x-5)(x-3)}$

The common denominator is $(x - 5)(x - 3)$

$$\frac{x+1}{x-5} - \frac{5}{x-3} = \frac{(x+1)(x-3) - 5(x-5)}{(x-5)(x-3)}$$

Expand the numerator: $x^2 - 7x + 22$

$$\frac{x^2 - 7x + 22}{(x-5)(x-3)}$$

Answer: $\frac{x^2 - 7x + 22}{(x-5)(x-3)}$

9 Bounds of speed, density and pressure

A train travels a distance of 3500 km, correct to the nearest 50.
The journey takes 425 hours, correct to the nearest 5.
Work out the lower bound of the speed.
Give your answer correct to 3 significant figures. (Calculator)

$3475 \leq$ distance < 3525; $422.5 \leq$ time < 427.5

Lower bound of speed = smallest distance ÷ biggest time = 3475 ÷ 427.5

= 8.12865... = 8.13 (3 s.f.)

Answer: 8.13 km/h

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10 Tangent to a circle

A circle has equation $x^2 + y^2 = 289$.
The point P(−15, −8) lies on the circle.
(a) Find the equation of the tangent to the circle at P. Give your answer in the form ax + by = c, where a, b and c are integers.
(b) The tangent meets the x-axis at the point T. Write down the coordinates of T.

(a) The gradient of the radius OP is $\frac{8}{15}$

The tangent is perpendicular to the radius, so its gradient is $-1 \div \frac{8}{15} = -\frac{15}{8}$

$y - (-8) = (-\frac{15}{8})(x - (-15))$

Multiply out and rearrange: $15x + 8y = -289$
(b) On the x-axis y = 0, so $15x = -289$
$x = -\frac{289}{15}$, so T is $(-\frac{289}{15}, 0)$

Answer: (a) $15x + 8y = -289$ (b) T = $(-\frac{289}{15}, 0)$