Day 65 **S Tier 10-a-day: Model Solutions**

1 Error intervals

The capacity of a tank is 235 litres, correct to the nearest 5.
Write down the error interval for the capacity of a tank, using c.

The value has been rounded to the nearest 5
Half of this is 2.5, so go 2.5 below and 2.5 above
235 − 2.5 = 232.5 and 235 + 2.5 = 237.5
$232.5 \leq c < 237.5$
Answer: $232.5 \leq c < 237.5$

2 Area of a triangle using sine

In triangle PQR, PQ = 20 cm and angle PQR = 30°.
The area of the triangle is 60 cm^2.
Work out the length of QR.
Do not use a calculator.

Area = $\frac{1}{2} \times PQ \times QR \times \sin Q$

$60 = \frac{1}{2} \times 20 \times QR \times \sin 30°$

$\sin 30° = \frac{1}{2}$, so $60 = 5 \times QR$

QR = 60 ÷ 5 = 12 cm
Answer: 12 cm

3 Distance-time graphs

The graph shows Hassan's cycle ride to a café and back.
(a) Work out Hassan's speed on the way back.
(b) Work out Hassan's average speed for the whole ride, including the stop.

(a) Return: 10 km in 1 hours, so speed = 10 ÷ 1 = 10 km/h
(b) Total distance = 2 × 10 = 20 km, total time = 2.5 hours
Average speed = 20 ÷ 2.5 = 8 km/h
Answer: (a) 10 km/h (b) 8 km/h

4 Algebraic proof

Prove that the product of (5n − 1) and (5n + 1) is always 1 less than a multiple of 25.

$(5n - 1)(5n + 1) = 25n^2 + 5n - 5n - 1$
$= 25n^2 - 1$
$25n^2$ is a multiple of 25, so the product is 1 less than a multiple of 25
Answer: Proved

5 Compound interest

Nadia invests £11,500 at 2% per year compound interest.
After how many whole years will the investment first be worth more than £13,300?
(Calculator)

Multiplier = 1.02
After 7 years: $11500 \times 1.02^7 = 13209.89$
After 8 years: $11500 \times 1.02^8 = 13474.08$
13474.08 is the first value above £13,300
Answer: 8 years

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6 Transformations of graphs

The point P(12, −1) lies on the graph of y = f(x).
Write down the coordinates of the image of P on the graph of
(a) y = f(x) − 3 (b) y = f(x − 3)

(a) y = f(x) − 3 is a translation of 3 down: (12, −4)
(b) y = f(x − 3) is a translation of 3 to the right: (15, −1)
Answer: (a) (12, −4) (b) (15, −1)

7 Standard form

Work out $(6.5 \times 10^7) \div (2.5 \times 10^5)$
Give your answer in standard form.

$6.5 \div 2.5 = 2.6$
$10^7 \div 10^5 = 10^2$
$= 2.6 \times 10^2$
Answer: 2.6×10^2

8 Speed and a moving train

A train travels at a constant speed of 90 km/h.
It takes 20 seconds to cross completely over a bridge which is 140 m long.
Work out the length of the train, in metres.

Speed = 90 km/h = 90,000 ÷ 3600 = 25 m/s
Distance travelled in 20 s = 25 × 20 = 500 m
This is the length of the bridge plus the length of the train
Train = 500 − 140 = 360 m
Answer: 360 m

9 Cone made from a sector

A sector of a circle has radius 20 cm and angle 180°.
The sector is made into a cone by joining the two straight edges together.
(a) Work out the radius of the base of the cone.
(b) Work out the exact volume of the cone. Give your answer in terms of π, as a surd if necessary.

(a) Arc length of the sector = $\frac{180}{360} \times 2\pi \times 20 = 20\pi$ cm
This is the circumference of the base: $2\pi r = 20\pi$, so r = 10 cm
(b) Slant height = 20 cm, so height$^2 = 20^2 - 10^2 = 300$, height = $10\sqrt{3}$ cm
Volume = $\frac{1}{3} \times \pi \times 10^2 \times 10\sqrt{3} = \frac{1000\sqrt{3}}{3}\pi$ cm^3
Answer: (a) 10 cm (b) $\frac{1000\sqrt{3}}{3}\pi$ cm^3

10 Algebraic proof (harder)

Prove that the difference between the cubes of any two consecutive integers is always an odd number.

Let the integers be m and m + 1.
$(m + 1)^3 - m^3 = m^3 + 3m^2 + 3m + 1 - m^3 = 3m^2 + 3m + 1$
$= 3m(m + 1) + 1$
m(m + 1) is even, so 3m(m + 1) is even, and adding 1 gives an odd number
Answer: Proved