Day 64 **S Tier 10-a-day: Model Solutions**

1 Averages with algebra

Here are five numbers: 2x, x + 2, 2x + 2, 3x, x.
The mean of the five numbers is 17.
(a) Work out the value of x.
(b) Work out the range of the five numbers.

(a) Total = (2x) + (x + 2) + (2x + 2) + (3x) + (x) = 17 × 5 = 85
9x + 4 = 85, so 9x = 81 and x = 9
The numbers are 18, 11, 20, 27, 9
(b) Range = 27 − 9 = 18
Answer: (a) x = 9 (b) 18

2 Equations with fractions

Solve $\frac{x+1}{3} + \frac{x+5}{5} = 4$

Multiply every term by 15: 5(x + 1) + 3(x + 5) = 60
Expand: 8x + 20 = 60
8x = 40
x = 5
Answer: x = 5

3 Surds

Given that $\sqrt{125} + \sqrt{45} = \sqrt{n}$,
find the value of n.

$\sqrt{125} = 5\sqrt{5}$ and $\sqrt{45} = 3\sqrt{5}$
$5\sqrt{5} + 3\sqrt{5} = 8\sqrt{5}$
$8\sqrt{5} = \sqrt{320}$, so n = 320
Answer: n = 320

4 Enlargement

Triangle P has vertices A(2, 3), B(3, 2) and C(−1, 2).
Triangle Q has vertices A'(1, 3), B'(3, 1) and C'(−5, 1).
Q is an enlargement of P.
Describe fully the single transformation that maps P onto Q.

x-step from A to B = 1, x-step from A' to B' = 2
Scale factor = 2 ÷ 1 = 2
The centre C satisfies: C + 2(A − C) = A', so (1 − 2)C = A' − 2 A
x: (−1) × x = 1 − 2 × 2 = −3, so x = 3
y: (−1) × y = 3 − 2 × 3 = −3, so y = 3
Answer: Enlargement, scale factor 2, centre (3, 3)

5 Profit and loss

A trader buys a guitar and adds a 25% mark-up to get the selling price.
In a sale the selling price is reduced by 30% to £280.
Work out how much the shop paid for a guitar.

Overall multiplier = 1.25 × 0.7 = 0.875
Cost = £280 ÷ 0.875
= £320
Answer: £320

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6 Iteration

Use the iteration formula $x_{n+1} = 3 + \frac{3}{x_n}$ with $x_0 = 1$ (Calculator)
to work out x_1, x_2 and x_3.
Give each answer correct to 3 decimal places.

$x_1 = 3 + \frac{3}{1} = 6.00000...$

$x_2 = 3 + \frac{3}{x_1} = 3.50000...$

$x_3 = 3 + \frac{3}{x_2} = 3.85714...$

Rounded: $x_1 = 6.000$, $x_2 = 3.500$, $x_3 = 3.857$

Answer: $x_1 = 6.000$, $x_2 = 3.500$, $x_3 = 3.857$

7 Moving averages

The table shows the monthly sales (thousands) over six periods.
The 3-point moving average centred on Mar is 17.
(a) Work out the value of x.
(b) Work out the 3-point moving average centred on Apr.

(a) $(14 + x + 25) \div 3 = 17$
$14 + x + 25 = 51$, so $x = 12$
(b) $(12 + 25 + 14) \div 3 = 51 \div 3 = 17$
Answer: (a) x = 12 (b) 17

8 Density of an alloy

An alloy contains 89 g of copper and 452 g of lead.
The density of copper is 8.9 g/cm^3. The density of lead is 11.3 g/cm^3.
Work out the density of the alloy.

Volume of copper = $89 \div 8.9 = 10$ cm^3
Volume of lead = $452 \div 11.3 = 40$ cm^3
Total mass = 541 g, total volume = 50 cm^3
Density = $541 \div 50 = 10.82$ g/cm^3
Answer: 10.82 g/cm^3

9 Perimeter and area of a running track

The diagram shows a running track made from a rectangle and two semicircles. (Calculator)
An athlete runs 5 complete laps around the track.
Work out the total distance run, to the nearest 10 metres.
The diagram is not drawn accurately.

Boundary of one lap = $2 \times 40 + 2 \times \pi \times 35 = 299.91$ m
5 laps = $5 \times 299.91 = 1499.6$ m
= 1500 m to the nearest 10 m
Answer: 1500 m

10 Tangency conditions

The line $y = kx - 3$ is a tangent to the curve $y = x^2 - 2$.
Find the possible values of k.
Give any surds in their simplest form.

Equate: $x^2 - 2 = kx - 3$
Rearrange: $x^2 - kx + 1 = 0$
One point of contact, so $b^2 - 4ac = 0$: $k^2 - 4 \times 1 = 0$
$k^2 = 4$
$k = \pm\sqrt{4} = \pm 2$
Answer: k = ±2