Day 63 **S Tier 10-a-day: Model Solutions**

1 Chords and Pythagoras in circles

AB is a chord of a circle, centre O. M is the midpoint of AB. (Calculator)
(a) Work out the length of the chord AB.
(b) Work out angle AOB, to 1 decimal place.
The diagram is not drawn accurately.

(a) $AM^2 = OA^2 - OM^2 = 29^2 - 20^2 = 441$
AM = 21 cm
AB = 2 × 21 = 42 cm

(b) sin(angle AOM) = $\frac{21}{29}$

Angle AOM = 46.40°, so angle AOB = 2 × 46.40 = 92.8°
Answer: (a) 42 cm (b) 92.8°

2 Stratified sampling

The table shows the number of members in each of four groups.
(a) A stratified sample of 31 is taken. Work out how many come from Group B.
(b) A different stratified sample has 18 from Group B. Work out the size of this sample.

Total = 200 + 120 + 140 + 160 = 620

(a) Group B: $\frac{120}{620} \times 31 = 6$

(b) Fraction sampled = 18 ÷ 120 = $\frac{3}{20}$, so sample size = $\frac{3}{20} \times 620 = 93$

Answer: (a) 6 (b) 93

3 Factorising fully

Factorise fully $5x^3 - 180x$

Take out the common factor 5x: $5x(x^2 - 36)$
$x^2 - 36$ is a difference of two squares: $(x - 6)(x + 6)$
Answer: $5x(x - 6)(x + 6)$

4 Weighted percentages

The ratio of juniors to seniors in a school is 1 : 4.
25% of the juniors and 30% of the seniors play a sport.
What percentage of all of these students play a sport?

Imagine 100 students in total: 20 juniors and 80 seniors
25% of 20 = 5 and 30% of 80 = 24
Together: 29 out of 100
29 ÷ 100 × 100 = 29%
Answer: 29%

5 Fractional indices

Given that x > 0, solve $x^{\frac{5}{4}} = 32$

Raise both sides to the power $\frac{4}{5}$

$x = (32)^{\frac{4}{5}}$
x = 16
Answer: x = 16

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6 Tangent to a circle

A circle has equation $(x + 4)^2 + (y + 2)^2 = 130$.
The point P(3, −11) lies on the circle.
Find an equation of the tangent to the circle at P.
Give your answer in the form ax + by = c, where a, b and c are integers.

The centre is (−4, −2). The gradient of the radius to P is $\frac{-11 - (-2)}{3 - (-4)} = -\frac{9}{7}$

The tangent is perpendicular to the radius, so its gradient is $\frac{7}{9}$. Using P: $y + 11 = \frac{7}{9}(x - 3)$

Multiply by 9: $9(y + 11) = 7(x - 3)$
Expand and rearrange: $-7x + 9y = -120$
Multiply by −1: $7x - 9y = 120$
Answer: $7x - 9y = 120$

7 Similar shapes

Two solid shapes, A and B, are mathematically similar.
The surface area of A is 24 cm^2 and the surface area of B is 216 cm^2.
The volume of A is 100 cm^3.
Work out the volume of B.

Ratio of areas = 24 : 216 = 1 : 9
Ratio of lengths = $\sqrt{1} : \sqrt{9}$ = 1 : 3
Ratio of volumes = $1^3 : 3^3$, so scale factor = 27
Volume of B = 100 × 27 = 2700 cm^3
Answer: 2700 cm^3

8 HCF and LCM

The HCF of two numbers is 3 and their LCM is 330.
One of the numbers is 33.
Work out the other number.

HCF × LCM = product of the two numbers
3 × 330 = 990
Other number = 990 ÷ 33 = 30
Answer: 30

9 Areas of similar triangles

In the diagram, DE is parallel to BC. AD = 9 cm, DB = 3 cm and the area of triangle ADE is 36 cm^2.
Work out the area of the trapezium DBCE.
The diagram is not drawn accurately.

Triangles ADE and ABC are similar. AB = 9 + 3 = 12 cm

Length scale factor = 12 ÷ 9 = $\frac{4}{3}$

Area scale factor = $(\frac{4}{3})^2 = \frac{16}{9}$

Area of triangle ABC = $36 \times \frac{16}{9}$ = 64 cm^2

Trapezium = 64 − 36 = 28 cm^2
Answer: 28 cm^2

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10 Exponential graphs

The diagram shows the graph of $y = ka^x$, where k and a are constants and $a > 0$.
The curve passes through the points (2, 24) and (4, 96).
Find the value of k and the value of a.

Substitute the points: $k\,a^2 = 24$ and $k\,a^4 = 96$

Divide: $a^2 = \frac{96}{24} = 4$

$a = 2$ (positive, since $a > 0$)

From the first point: $k \times 4 = 24$, so $k = 6$

Answer: k = 6, a = 2