Day 62 **S Tier 10-a-day: Model Solutions**

1 Tests of divisibility

The 4-digit number 50k9 is a multiple of 3.
k is a single digit.
Find all the possible values of k.

A number is a multiple of 3 if its digit sum is a multiple of 3
Digit sum = 5 + 0 + 9 + k = 14 + k
This must be a multiple of 3, so k = 1 or 4 or 7
Answer: k = 1 or 4 or 7

2 Vectors in a triangle

OAB is a triangle. OA = $\mathbf{a}$ and OB = $\mathbf{b}$.
M is on OA with OM : MA = 1 : 3. N is on AB with AN : NB = 1 : 2.
(a) Find ON in terms of $\mathbf{a}$ and $\mathbf{b}$.
(b) Find MN in terms of $\mathbf{a}$ and $\mathbf{b}$.
The diagram is not drawn accurately.

(a) AB = $-\mathbf{a} + \mathbf{b}$, so AN = $\frac{1}{3}(\mathbf{b} - \mathbf{a})$

ON = OA + AN = $\mathbf{a} + \frac{1}{3}(\mathbf{b} - \mathbf{a}) = \frac{2}{3}\mathbf{a} + \frac{1}{3}\mathbf{b}$

(b) OM = $\frac{1}{4}\mathbf{a}$

MN = ON − OM = $\frac{2}{3}\mathbf{a} + \frac{1}{3}\mathbf{b} - \frac{1}{4}\mathbf{a} = \frac{5}{12}\mathbf{a} + \frac{1}{3}\mathbf{b}$

Answer: (a) $\frac{2}{3}\mathbf{a} + \frac{1}{3}\mathbf{b}$ (b) $\frac{5}{12}\mathbf{a} + \frac{1}{3}\mathbf{b}$

3 Laws of indices with algebra

Simplify fully $\frac{4x^4y^6 \times 5xy^6}{2x^4y^3}$

Give your answer in the form kx^ay^b

Numbers: 4 × 5 ÷ 2 = 10
Powers of x: 4 + 1 − 4 = 1
Powers of y: 6 + 6 − 3 = 9
= $10xy^9$
Answer: $10xy^9$

4 Laws of indices

Given that $\frac{27^2 \times 9^2}{9} = 3^n$,

find the value of n.

27 = 3^3, 9 = 3^2
Numerator: $3^6 \times 3^4 = 3^{10}$
Denominator: 3^2
Subtract the powers: 10 − 2 = 8
Answer: n = 8

5 Pressure

A metal crate in the shape of a cuboid weighs 1170 N.
Its smallest face has area 1200 cm^2. Its largest face has area 3,000 cm^2.
Pressure = force ÷ area
Work out (a) the greatest pressure and (b) the least pressure the metal crate can exert on the ground, in N/m^2.

1 m^2 = 10 000 cm^2, so 1200 cm^2 = 0.12 m^2 and 3,000 cm^2 = 0.3 m^2
(a) Greatest pressure is on the smallest face: 1170 ÷ 0.12 = 9,750 N/m^2
(b) Least pressure is on the largest face: 1170 ÷ 0.3 = 3,900 N/m^2
Answer: (a) 9,750 N/m^2 (b) 3,900 N/m^2

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6 Direct and inverse proportion

V is inversely proportional to r.
When r = 6, V = 2.
(a) Find a formula for V in terms of r.
(b) Find the value of V when r = 3.

(a) $V = \frac{k}{r}$

Substitute r = 6 and V = 2: $2 = \frac{k}{6}$

k = 12

$V = \frac{12}{r}$

(b) $V = \frac{12}{3} = 4$

Answer: (a) $V = \frac{12}{r}$ (b) 4

7 Radius from chord and height of arc

A circular arc has a chord AB of length 12 cm.
The greatest distance between the chord and the arc is 2 cm.
Work out the radius of the circle.

Let the radius be r. The centre O is on the line through the midpoint M of AB and the highest point of the arc, so OM = r − 2

AM = 12 ÷ 2 = 6. Pythagoras in triangle OMA: $r^2 = (r - 2)^2 + 6^2$

$r^2 = r^2 - 4r + 4 + 36$

4r = 40, so r = 10 cm

Answer: 10 cm

8 Ratio and algebra

In a club, the ratio of the number of boys to the number of girls is 6 : 5.
Then 17 more boys are added and 15 more girls are added.
The ratio is now 7 : 6.
How many girls were there at the start?

Let the numbers at the start be 6k and 5k
(6k + 17) : (5k + 15) = 7 : 6, so 6(6k + 17) = 7(5k + 15)
36k + 102 = 35k + 105
1k = 3, so k = 3
Number of girls at the start: 5 × 3 = 15
Answer: 15

9 Circle and line intersections

Solve the simultaneous equations
$x^2 + y^2 = 58$
y = x − 4

Substitute into the first equation: $x^2 + (x - 4)^2 = 58$

Expand: $2x^2 - 8x - 42 = 0$

Divide by 2: $x^2 - 4x - 21 = 0$

Factorise: (x + 3)(x − 7) = 0, so x = −3 or x = 7

y = x − 4 gives y = −7 and y = 3

Answer: x = −3, y = −7 and x = 7, y = 3

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10 Angles with algebra

In triangle PQR, side QR is extended to S.
Angle QPR = (4x + 25)°
Angle PQR = (2x − 15)°
Angle PRS = (4x + 56)°
(a) Work out the value of x.
(b) Work out the size of angle PRQ.

(a) The exterior angle of a triangle equals the sum of the two opposite interior angles
4x + 56 = 6x + 10
2x = 46
x = 23
(b) Angle PRS = 148° so angle PRQ = 180° − 148° = 32°
Answer: (a) x = 23 (b) 32°