Day 61 **S Tier 10-a-day: Model Solutions**

1 Mean of a changing set

The mean of 5 numbers is 52.
One number is removed and the mean of the remaining 4 numbers is 55.5.
Work out the number that was removed.

Total of the 5 numbers = 5 × 52 = 260
Total of the 4 remaining = 4 × 55.5 = 222
Number removed = 260 − 222 = 38
Answer: 38

2 Fractions of fractions

Hana read $\frac{2}{7}$ of a book on Saturday.

On Sunday she read $\frac{7}{8}$ of the pages that were left.

She read 105 pages on Sunday.
How many pages are in the book?

The second amount is $\frac{7}{8}$ of $\frac{5}{7}$, which is $\frac{5}{8}$ of the total

$\frac{5}{8}$ of the total is 105

Total = $105 \div \frac{5}{8} = 168$

Answer: 168 pages

3 Trigonometric graphs

The diagram shows the graph of y = a cos bx, where a and b are positive integers.
(a) Write down the value of a and the value of b.
(b) Work out the value of y when x = 150°.

(a) The maximum value is 4 and the minimum is −4, so a = 4
The graph completes 4 full cycles between 0° and 360°, so b = 4
(b) y = 4 cos (4 × 150°) = 4 cos 600°

cos 240° = $-\frac{1}{2}$, so y = −2

Answer: (a) a = 4, b = 4 (b) y = −2

4 Quadratic graphs

The curve $y = x^2 - 2x - 3$ crosses the x-axis at the points A and B.
(a) Find the coordinates of A and B.
(b) Find the coordinates of the minimum point of the curve.

(a) Put y = 0: $x^2 - 2x - 3 = 0$, so (x + 1)(x − 3) = 0
x + 1 = 0 or x − 3 = 0, so x = −1 or x = 3
(b) The turning point is halfway between A and B: x = (−1 + 3) ÷ 2 = 1
When x = 1: $y = 1^2 - 2 \times 1 - 3 = -4$
Answer: (a) A(−1, 0), B(3, 0) (b) (1, −4)

5 Pressure

A metal crate in the shape of a cuboid weighs 180 N.
Its smallest face has area 300 cm^2. Its largest face has area 1,000 cm^2.
Pressure = force ÷ area
Work out (a) the greatest pressure and (b) the least pressure the metal crate can exert on the ground, in N/m^2.

1 m^2 = 10 000 cm^2, so 300 cm^2 = 0.03 m^2 and 1,000 cm^2 = 0.1 m^2
(a) Greatest pressure is on the smallest face: 180 ÷ 0.03 = 6,000 N/m^2
(b) Least pressure is on the largest face: 180 ÷ 0.1 = 1,800 N/m^2
Answer: (a) 6,000 N/m^2 (b) 1,800 N/m^2

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6 Alternate segment theorem

PAQ is a tangent to the circle at A. B and C are points on the circle. Angle PAB = 62° and angle CAQ = 51°.
(a) Work out the size of angle ABC.
(b) Work out the size of angle ACB.
The diagram is not drawn accurately.

(a) Angle ABC = angle CAQ = 51° (alternate segment theorem)
(b) Angle ACB = angle PAB = 62° (alternate segment theorem)
Answer: (a) 51° (b) 62°

7 Equation of a curve from a sketch

The diagram shows a sketch of a quadratic curve. It crosses the axes at the points shown.
Find an equation of the curve.
Give your answer in the form $y = ax^2 + bx + c$.

The roots are $x = -1$ and $x = 3$, so $y = a(x + 1)(x - 3)$
When $x = 0$, $y = -3$: $a \times 1 \times (-3) = -3$, so $a = 1$
$y = (x + 1)(x - 3)$
$y = x^2 - 2x - 3$
Answer: $y = x^2 - 2x - 3$

8 Reverse percentages

The price of a tablet is increased by 30%.
Later the new price is reduced by 10% in a sale.
The sale price is £468.
Work out the price before the increase. (Calculator)

Multipliers: 1.3 and 0.9
Overall multiplier = 1.3 × 0.9 = 1.17
Original × 1.17 = £468
Original = £468 ÷ 1.17 = £400
Answer: £400

9 Tangent to a circle

A circle has equation $x^2 + y^2 = 289$.
The point P(15, 8) lies on the circle.
(a) Find the equation of the tangent to the circle at P. Give your answer in the form ax + by = c, where a, b and c are integers.
(b) The tangent meets the x-axis at the point T. Write down the coordinates of T.

(a) The gradient of the radius OP is $\frac{8}{15}$

The tangent is perpendicular to the radius, so its gradient is $-1 \div \frac{8}{15} = -\frac{15}{8}$

$y - 8 = (-\frac{15}{8})(x - 15)$

Multiply out and rearrange: $15x + 8y = 289$
(b) On the x-axis $y = 0$, so $15x = 289$
$x = \frac{289}{15}$, so T is $(\frac{289}{15}, 0)$

Answer: (a) $15x + 8y = 289$ (b) T = $(\frac{289}{15}, 0)$

10 Squares, cubes and prime factors

Find the smallest positive integer k such that 980 × k is a square number.

$980 = 2^2 \times 5 \times 7^2$
For a square number every power must be a multiple of 2
Multiply by 5 to get $2^2 \times 5^2 \times 7^2$
k = 5
Answer: k = 5